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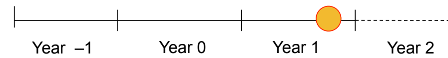
Project ℤ
Mapping Development Trajectories of Students' Conceptions of Integers

Solve the problem: 3 – 5 = ☐

Solve the problem: 3 – ☐ = -2

Solve the problem: -5 + -1 = ☐

3-Year Project (2009–2012)



Research Questions

1. What are Grades 2, 4, 7, and 11 students' conceptions of integers and operations on integers?
2. What are possible trajectories of students' ways of reasoning about integers?

Why Study Integers?

- When compared with the literature on rational numbers or place value, literature on students' understanding of integers is relatively sparse (NRC, 2001).
- Students often have great difficulty operating on integers, and those difficulties appear to be robust (see, e.g., Thomaidis & Tzanakis, 2007; Vlassis, 2002). Even those who have completed algebra courses are challenged by problems with negative numbers (Reck & Mora, 2004; Vlassis, 2002).
- Integers mark a transition from arithmetic to algebra because of their abstract nature (negative numbers have no concrete embodiments) and because students must understand fundamental algebraic principles, for example, using additive inverses, which first come into play with the introduction of integers.
- Difficulties in algebra have been linked to a lack of integer understanding (see e.g., Moses, 1989).

Methods

- Developed integers interview appropriate for Grades 1–12 students and piloted more than 90 interviews.
- Conducted 160 problem-solving interviews using a cross-sectional design.
  - 40 from each of Grades 2, 4, 7, and 11 across 11 ethnically diverse schools with varying API scores in San Diego County. (Grade 11 students were enrolled in Precalculus or Calculus.)
  - Problem-solving interviews lasted about 1.5 hours.
  - Interview tasks included open number sentences, comparison problems, and context problems—the majority of items were open number sentences.
- A coding scheme for interview-tasks data was developed using the constant-comparative method.
  - The coding scheme includes 5 broad categories of students' integer reasoning: ordinal reasoning, analogically based reasoning, alternative/limited reasoning, computationally based reasoning, and formal-mathematical reasoning.

Interview Tasks

Open Number Sentences	Comparison Tasks	Context Problems
Problems of the form: 6 – 11 = ☐ 12 + ☐ = 4 5 – ☐ = 8 -5 – -2 = ☐ ☐ – 6 = -3 -9 + ☐ = -4	For each pair, circle the larger, write "=" if the two quantities are equal, or write "?" if there is not enough information to determine which is larger. a) 3 -7 b) 0 -9 c) -5 -6 d) -100 -5 e) - -3 -3 f) ☐ – ☐	Yesterday you borrowed $8 from your friend to buy a school t-shirt. Today you borrowed another $5 from the same friend. Does your friend owe you money, do you owe your friend money, or is it some other situation? Can you write an equation or number sentence that describes this story and explain how it relates to the story?

Ways of Reasoning About Integers

Violet: Order-based Reasoning

The child imposes an ordering on ℤ, and uses the positional (or ordinal) nature of numbers to solve problems. Strategies include counting, use of motion on the number line, and jumping to zero.

Number Line 3 – ☐ = -2
Violet placed her pencil at "3" on the number line. She said that she knew to go "backwards" (to the left) because the problem was a subtraction problem. She counted back on the number line, counting by ones (1, 2, 3, 4, 5), until she landed at -2. Her answer was 5.

Jorge: Analogically-based Reasoning

Responses supported by comparison to other entities that the student deems to be, in some way, structurally similar to negative integers. For example, the student may treat negative integers like positive integers, negative charges, or dollars owed.

Negatives Like Positives -5 + -1 = ☐
Jorge answered that -5 + -1 equals -6 because he ignored the negative sign and added 5 and 1 to get 6. He knew that, in this case, negative numbers behave similarly to positive numbers and so his answer would have to be -6.

Findings Sampler

1a. PREDICT

What percentage of 2nd, 4th, 7th, and 11th-grade students correctly responded to **5 – ☐ = 8**?

1b. RESULTS

Grade	2 (n = 13)	4 (n = 28)	7 (n = 40)	11 (n = 40)
% correct	8%	4%	50%	90%

1c. COMMENTARY

Across every grade level, this problem was the most challenging of all the open number sentences given in the interview. Why might this problem represent such difficulty for students?

1d. ONE EXPLANATION

At least one student in every grade viewed this problem as not possible because, from their perspective, one cannot get a difference that is larger than the minuend. That is, they expressed the overgeneralization that subtraction makes smaller. This way of reasoning is one of the *Limited Conceptions* we categorized in our work. We call open number sentences such as the one in 1a counterintuitive because they directly contradict the idea that subtraction makes smaller.

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2a. PREDICT

What percentage of 2nd, 4th, 7th, and 11th-grade students correctly responded to **-8 – ☐ = -2**?

2b. RESULTS

Grade	2 (n = 13)	4 (n = 28)	7 (n = 40)	11 (n = 40)
% correct	31%	61%	53%	98%

2c. COMMENTARY

Students in grades 2 and 4 were much more successful with this problem than with Problem 1, and 4th-grade students (who had received *no* formal integers instruction) outperformed 7th-grade students (who had received integers-related instruction). How could it be that 4th-grade students outperformed 7th-grade students?

2d. ONE EXPLANATION

At every grade level, students demonstrated appropriate use of an analogically based way of reasoning we call *Negatives like Positives*. Because 2nd- and 4th-grade students had no rules to try to remember, they often adopted this way of reasoning. For example, a child would say, "I know that 8 minus 6 is 2, so -8 minus -6 is -2." Young students in particular invoked this way of reasoning (*Negatives like Positives*) for problem 2a.