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Students' Integer Reasoning: Results From a Large-Scale Study

We share findings from a large-scale study of 160 K–12 students' ways of reasoning about integer addition and subtraction during individual problem-solving interviews. Findings are from students' solutions to open number sentences (decontextualized problems such as -3 + □ = 6) and include

- Young children can reason productively about negative numbers. We found that ½ of all 2nd and 4th graders in our study had heard of negative integers *before* any school-based integer instruction, and ¾ of 2nd and 4th graders were able to solve at least 1 integer problem correctly.
- We have identified five broad categories of reasoning that K–12 students use when solving open number sentences. Across grade levels, the patterns of use and frequency with which different ways of reasoning are used vary (see chart below).

Ways of Reasoning	**Definition**	**Ways of Reasoning % Use (by total # of problems)**			
		2/4 No[a]	*2/4 Yes*	*7th*	*11th*
Order	Using the sequential and ordered nature of numbers to reason about a problem (e.g., counting strategies or a number line with motion).	0%	33%	38%	19%
Analogical	Relating negative numbers to another idea/concept and reasoning about negative numbers on the basis of behaviors observed in this other concept. Negative numbers may be related to a countable amount or quantity and tied to ideas about cardinality/magnitude, or they may be related to contexts.	0%	11%	20%	16%
Formal	Treating negative numbers as formal objects that exist in a system and are subject to fundamental mathematical principles that govern behavior. Formal strategies often involve comparisons to other, known, problems so that the logic of the approach remains consistent and underlying structural principles are not violated.	0%	3%	12%	24%
Computational	Using a procedure, rule, or calculation to arrive at an answer.	13%	13%	53%	75%
Alternative	Using strategies that reflect incomplete or limited views of negative numbers and may have invalid mathematical foundations. At times, the domain of possible solutions is locally restricted to **W**.	93%	59%	14%	<1%

Note. Because students can use more than one way of reason to solve a problem, column-percentage sums are larger than 100%. [a]Students without negative numbers in their number domains.

- Problem types are important, and are important in different ways for different grade levels. For example, before integer instruction, -5 – -3 = □ and -5 + -1 = □ are the easiest problems for 2nd/4th-grade children (87% correct each), yet 6 + -3 = □ and 6 – -2 = □ are among the harder questions for them (13% correct each). And, 2nd and 4th graders do *better* than 7th graders on -5 – -3 = □ and -5 + -1 = □ (78% and 83% correct, respectively, for 7th grade). Our data enable us to account for these results on the basis of the ways of reasoning students used. We believe that way of reasoning, problem type, and grade level interact, and we are in the process of analyzing these interactions.
- *Flexibility* is a measure of the variety of ways of reasoning (WoR) used to solve integer tasks; it indicates whether a student uses primarily one WoR or chooses different WoR depending on the affordances of the problem. We found that flexibility increases when we move up grade levels and is positively correlated with performance in our data, both across grades ($r = .6$) and within grades ($r = .36, .40, .34$ for 2/4, 7th, and 11th, respectively).

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[1]Project Z personnel include Lisa Lamb, Randy Philipp, Ian Whitacre, Bonnie Schappelle, Spencer Bagley, Mindy Lewis & Jessica Bishop.

Implications for Future Research

We see two primary directions for future research that build on our work: (a) continued research on students' integer conceptions (e.g., longitudinal studies, studies using different sets of integer tasks, or further exploration/extension of WoR), and (b) research on teaching/ teachers while teachers use this knowledge to inform integer instruction.

Additional Reading

Bishop, J. P., Lamb, L. C., Philipp, R. A., Whitacre, I., Schappelle, B. P., & Lewis, M. L. (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. *Journal for Research in Mathematics Education, 45*(1), 19–61.

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Whitacre, I., Bishop, J. P., Lamb, L. C., Philipp, R. A., Schappelle, B. P., & Lewis, M. (2012). Happy and sad thoughts: An exploration of children's integer reasoning. *Journal of Mathematical Behavior*, 31, 356–365.

Whitacre, I., Bishop, J. P., Lamb, L. C., Philipp, R. A., & Schappelle, B. P., & Lewis, M. (in press). Dollars and Sense: Students' Integer Perspectives. Accepted for publication in *Mathematics Teaching in the Middle School.*

Visit the Project Z Website,
http://www.sci.sdsu.edu/CRMSE/projectz/index.html

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