Project Z
Literature Overview
What clusters of research on integers would you identify?

Building on the Emerging Knowledge Base
for Teaching and Learning in Relation to Integers

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Overview of This Presentation

- Why did we do this?
  - It is important to know where the field is if you are interested in advancing knowledge

- Literature-search process
  - search terms, inclusion/exclusion criteria

- Our results: Clusters of research activity

- Tour of the clusters

Literature-Search Process

- Keywords: Integers, Negative Numbers
  - filtered to remove advanced mathematical topics

- Search engines: Education Full Text, ERIC, Education Retrospective

- 1508 records were coded, many duplicates

- After removing duplicates, records before 1980, and publications that were clearly unsuitable, 211 records remained
  - 47 of these were in research journals

Results

- Clusters of research activity
  - Examining fluency
  - Documenting conceptions
  - Improving student learning
  - Intersecting algebra and integers
  - Providing historical analyses
  - (Negatives illustrative)
  - (Think-pieces)

- By no means are these categories mutually exclusive

Clusters of Research Activity

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Examining Fluency

- Characteristics
  - o Large-n quantitative studies, usually assessing correctness
  - o Analyses of common errors—Deficit model
  - o Identifying misconceptions and difficulties

- Central paper
  - o Kloosterman (2010)—Summary of NAEP data

- Other papers
  - o Kaur, Sharon, & Huey (1994); Hativa & Cohen (1995); Bruno, Espinel, & Martinon (1997); Vlassis (2004); Lim (2010); Widjaja, Stacey, & Steinle (2011)

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Documenting Conceptions

- Characteristics
  - Building models of student thinking (preinstruction)
  - Analyses of *thinking* rather than *performance*

- Central papers
  - Chiu (1994)
  - Liebeck (1990)
- Other papers
  - Thompson & Dreyfus (1988); Mukhopadhyay (1997); Bruno & Martinon (1999); Linchevski & Williams (1999); Gallardo (2002); Vlassis (2004); Sfard (2007); Thomaidis & Tzanakis (2007)

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Improving Student Learning

- Characteristics
  - o Investigations of theoretically grounded ideas that may help students learn about negatives
  - o Often rooted in a context
  - o Usually includes a quantitative pre/post piece, often complemented with qualitative analysis of interview data

- Central papers
  - o Stephan & Akyuz (2012)—Net worth (Michelle will be presenting)
  - o Linchevski & Williams (1999)—Using double abacus in context of disco game

- Other papers
  - o Thompson & Dreyfus (1988); Hativa & Cohen (1995); Moreno & Mayer (1999); Vlassis (2002); Moreno & Duran (2004); Altiparmak & Ozdogan (2010); Saxe et al. (2010)

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Intersecting Algebra and Integers

- Characteristics
    - Many topics in algebra are influenced by understanding of integers and operations.
        - Many student problems with algebra stem from problems with negative numbers.
    - Examining these topics that lie in the intersection

- Central papers
    - Christou & Vosniadou (2012)—Literal symbols representing negative numbers
    - Vlassis (2002)—Solving equations

- Other papers
    - Kaur, Sharon, & Huey (1994); Sfard & Linchevski (1994); Thomaidis & Tzanakis (2007)

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Providing Historical Analyses

- Characteristics
  - o Analyses of historical documents to understand the evolution of the negative number concept
  - o Many couple a historical piece with an empirical piece, using the history to guide analysis

- Major, consistent findings
  - o The process of acceptance is lengthy.
  - o It is possible to compute using negative numbers before accepting their meaningfulness.
  - o Resigned acceptance was prompted by the advent of abstraction and formalism.

Providing Historical Analyses

- Many student difficulties are similar to difficulties faced by mathematicians.

Providing Historical Analyses

ଓଃ (Video of Max removed for size purposes.)

Providing Historical Analyses

- Many student difficulties are similar to difficulties faced by mathematicians.

- "I know some who cannot understand that to take four from nothing leaves nothing."

- ℛ Blaise Pascal, 1669, *Pensées* (as cited in Bishop et al., *Journal for Research in Mathematics Education*, *45*(1), 19–61)

Providing Historical Analyses

- Central papers
  - Hefendehl-Hebeker (1991)—in-depth analysis of primary sources
  - Bishop et al. (2014)—using historical data and children's thinking data together to identify obstacles and affordances

- Other papers
  - Hitchcock (1997); Gallardo (2002); Thomaidis & Tzanakis (2007)

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Less-Central Clusters

- Negatives illustrative
  - Could have been materially similar without examining negative numbers, even if negative numbers are the best illustration.
  - Choppin (2011)—Teacher moves
  - See also Bell (1993); Sfard & Linchevski (1994); Featherstone (2000); Sfard (2007); Thomaidis & Tzanakis (2007)

- Think-pieces
  - May provide theoretical rationale for proposed methods of teaching, reflect on classroom experience, etc.
  - Sfard (2007)—Commognition
  - Sfard & Linchevski (1994)—Reification
  - See also Streefland (1996); Featherstone (2000); Selter, Prediger, Nuhrenborger, & Hußmann (2012)

Summary

- Recap: Clusters of research activity
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