Integers Problem-Solving Interview

Directions to interviewer. Have the participant solve each problem, one at a time. For problems for which several items are on a page, use a blank sheet of paper to cover future problems so that the student sees and solves one problem at a time. The student can look back to see the problems he or she has solved but cannot see upcoming problems. Note that unless otherwise specified, problems are to be posed to students at all grade levels. Italicized text should be read aloud to the participant.

Materials: 20 unifix cubes, blank paper, student version of interview, number line with 20 tick marks to the left of 0 and tick marks and numbering from 0 to 20.

Be aware of your language! Use the child's language for negative and positive numbers. For example, do not use the term *negative* OR *positive* unless the child has used the terms. Young children may not have heard the term *positive* invoked to refer to natural numbers.

Regardless of students' comments, do not prompt the child to draw a number line. We do not want a premature conversation about the number line to influence responses to the remainder of the problems.

What do you do if the child operates solely within the domain of whole numbers? Pose 1–14 and then pose only 23–28.

Interview Introduction

I am working with others on a project, and we are trying to learn how students of different ages think about numbers.

I am going to ask you some questions, and the purpose of the questions is to learn how you are thinking. Some problems may seem easy and some may seem challenging. We do not expect you to solve every problem correctly. After every problem I will ask you how you thought about it, whether your answer is correct or incorrect. If you can tell me how you are thinking about the problem while you are solving it, that will help me. By sharing your thinking, you are helping me learn how to be a better teacher.

Before beginning the interview, have the videographer hold up a sign identifying the interviewer, child, grade level, date, and school site.

The following statements need to be captured on the videotape:
The date today is [day of week – Mon, Tue, etc.], [date] _____*, 2011. My name is* _____ *and I am interviewing* [name of child]. [Child's name] *is in* __ *grade at* _____ *school.*

For each of the following, we are tracking only the answers; no explanations are to be requested.

1. *Name a big number. Can you name a bigger number?*
2. *Name a small number. Can you name a smaller number?* If the child responds, "One," ask, *What if I gave that away. What number would you have then?* If the child responds, "Zero," ask, *Is there a number smaller than zero?*
3. *Can you count backward, starting at 5?* If the child stops at 0 or 1, ask, *Can you keep counting back?* (If the child continues to count back, have the child stop counting at -5).

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NOTE. For Grades 2 and 4 students, interviewer does not pose #4 unless the student has previously mentioned the term *negative.* The interviewer should not introduce the term *negative* or the notation for negative numbers unless the child mentions them in 1–3. Note that negative numbers are explicitly used in the open number sentences but initial sequencing is deliberate so as to not introduce the concept of negatives as possible solutions to the three counterintuitive problems.

4. *What can you tell me about negative numbers*?
   ONLY if the child is struggling to respond*: If you were to explain them to someone who is in 4th (or younger) grade, how would you explain them*

OPEN NUMBER SENTENCES

We pose 5–8 to establish a baseline regarding whether children can solve these types of problems over the domain of whole numbers. We next pose 9–12 as a group because the problem statements involve only nonnegative numbers. We delay the introduction of negative numbers until Item13 to learn how children will respond on their own, without interviewer introduction of negative numbers.

1. Please read each problem out loud so that we all know what problem you are solving.
[Pull the paper down so that the first problem is visible and point to the box, ☐.]
2. We are going to call this box. *In the box, you will write the number that you think makes the number sentence true.*
3. If you think that a problem does not have an answer or if you just don't know, you may say so.
4. You may use the number line, the unifix cubes, your fingers, or you may solve a problem in your head -- whatever you want, to help you solve the problems.

5. 5 + 6 = ☐ *How did you think about that?*
6. 4 + ☐ = 9 *How did you think about that?*
7. ☐ – 4 = 6 *How did you think about that?*
8. 8 – ☐ = 4 *How did you think about that?*

On all remaining problems, the follow-up is *How did you think about that?* If an unclear explanation is given, you might follow up with *Pretend that you have to explain that to a third (*younger*) grader. What would you say?*

Because we want to identify the strategy in addition to the way of reasoning, you <u>may also</u> need to probe: *I understand how you were thinking about that. I am curious about just how you got to your answer of* ___ or *I am curious how you chose the number* ___ . Be sure to use the child's response in the blank. We hope that this question will help us understand whether and how the child is counting, using a recalled fact, using a fact family, and so on. This latter code (i.e., strategy code) is of secondary importance, so you may not ask the "How did you get to your answer" question for every problem.

Do NOT ask for a second strategy (until #35—see prompt).

If the child states that a problem cannot be solved, have the child write "no" in the blank. If the child thinks that the problem has a solution but does not know what that solution is,

write "?" in the blank. If a problem is not posed, place a slash through the problem. This notation will help us with record keeping and data entry so that we know in which cases we posed the problem but the child thought that the problem had no solution.

For each of the problems that I am going to ask you, please read the problem aloud and write your answer in the box.

9. $3 - 5 = \square$

NOTE. If the child responds, "2," then **write**, *"*$5 - 3 = \square$." After the child gives a response of 2, then remark, *That's interesting. I notice that both of these problems have answers of 2. What do you think about that?*
**Also, if the child states that one can switch the order of the 3 and the 5, ask the child to show 5 – 3 with fingers or cubes. Then ask the child to show 3 – 5 with fingers or cubes to help determine whether child has negatives in his or her domain.

[If after this follow-up, the child changes the answer to 0, then use the question below.]
OR
If the child responds, "Zero," then ask, "*What is 3 – 4? What is 3 – 3? That is interesting. All the answers are zero. What do you think about that*?"

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10. $6 + \square = 4$

    If the child struggles to answer, ask, *What makes this problem hard?* (Note, try to identify whether AMB/SMS is a factor.) If the child notes that the problem can be solved *only* with a different operation (e.g., 6 + __ = 4 can be solved only if the problem was subtraction, 6 – ___ = 4), ask, *If we go back to the way the problem is written on the page, is there anything you can put in the box to make the number sentence true?*

11. $5 - \square = 8$

    If the child struggles to answer, ask, *What makes this problem hard?* (Note, try to identify whether AMB/SMS is a factor.) If the child notes that the problem can be solved *only* with a different operation (e.g., 5 –__= 8 can be solved only if the problem was addition, 5 +__ = 8), ask, *If we go back to the way the problem is written on the page, is there anything you can put in the box to box to make the number sentence true?*

12. $\square + 6 = 2$

    If the child struggles to answer, ask, *What makes this problem hard?* (Note, try to identify whether AMB/SMS is a factor.)

13. $-3 + 6 = \square$
    If the child seems to be treating –3 + 6 incorrectly as –(3 + 6), look for additional confirming or refuting evidence on #14.

14. $-8 - 3 = \square$ (Check to see whether the child is treating –8 – 3 incorrectly as – (8 –3)).

    NOTE. If the child does not appear to understand the notation for negative numbers or has not yet invoked negative numbers (i.e., reads problem 14 as "minus 8 minus 3"), you may follow-up saying, *Some children have read this problem as negative 8 minus 3. What do you think about that?*

Not possible = NP "I don't know" = ?
Problem not posed = diagonal slash through problem # (or entire page)

PROBLEM IN CONTEXT

15a. *Yesterday you borrowed $8 from your friend to buy a school t-shirt, and today you borrowed another $5 from the same friend to buy lunch. ** What's the situation now?*
***Do you still owe your friend money or does your friend owe you money? How much?*

The overriding point is that we need to get a dollar amount from the child. If young children are struggling, you may need to add the name of the child's friend and elaborate on the story.

15b. *Can you write an equation or number sentence that describes this story? Can you explain how this number sentence (equation) relates to the story?* [Be sure to ask child to explain the number sentence.]

15c. *Are there other equations that describe the story? If so, please write those equations. Can you explain how this number sentence (equation) relates to the story?* [Be sure to ask child to explain each number sentence.] If the child has given multiple equations, an optional question to pose is *How did you actually solve it?*

If at this point, the child has not recently referred to the context (but has been talking primarily about equations), you may need to ask, Can you remind me what the story is? [Feel free to ask this question at any time during this interaction.]

I am going to show you three different equations students have written for this problem. They may or may not match your equations.

**First, show the child the same equation that the child wrote for 15b, if the equation the child wrote is one of the equations written below (for most students, the first equation you show will be 15f). Keep the other equations covered.

15d. -8 + -5 = -13

15e. -8 – 5 = -13

15f. 8 + 5 = 13

After each equation ask the following questions:
I saw a child who wrote this equation (point to equation). Would this number sentence (equation) describe this story? Can you explain how this number sentence (equation) relates to the story?

**To have a written record of students' responses, have students write "yes" or "no" next to each equation in response to the above question.

If the child is considering only the domain of natural numbers, skip to problems 23–28. Note that problems 23–28 will reside as a set on a separate page in the student work packet.

Not possible = NP "I don't know" = ?
Problem not posed = diagonal slash through problem # (or entire page)

SKIP PROBLEMS ON THIS PAGE IF THE CHILD IS OPERATING SOLELY IN THE DOMAIN OF NATURAL NUMBERS. GO TO NEXT PAGE AND POSE OPEN-NUMBER-SENTENCE PROBLEMS 23–28.

16. $-2 + \square = 4$

17. $\square - 5 = -1$

18. $-9 + \square = -4$ *How determine sign?*

19. $-2 - \square = -8$ *How determine sign?*

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20. $-5 + \square = -8$ *How determine sign?*

21. $-3 - \square = 2$

22. $-8 - \square = -2$

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23. -8 + ☐ = 0

24. -5 + -1 = ☐

25. -5 – -3 = ☐

If the child uses the double switch, ask **both** questions below. If the child does **not** use the double switch, ensure that you understand their meaning for subtracting negatives.

If you were explaining to a younger child why you can change the problem like that, what would you say? If the student says, "Because my teacher told me," ask, *Does changing the signs always give a correct answer? Why do you think that it is always correct?*

(Below, be sure to use the child's language for *negative.*)
Do you have another way of thinking about what it means to subtract negative 3 from negative 5?

Note that this question is <u>not</u> about the equivalence but about coming up with an explanation for subtracting a negative number. Changing the problem to an addition problem is <u>not</u> another way.

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26. a) 6 – -2 = ☐

b) 6 – -2 = ☐ (use 26b **only** if the child invokes the **double switch**. If the child invokes the double switch, point to the clean version of the problem and state, *Here is the original problem. Before you changed the problem, was the answer to the original problem [answer to 26a]?* (If necessary, ask, *So, do you have a way of thinking about what it means to subtract negative 2 from 6?* (Note that this last question in particular is about coming up with an explanation for subtracting a negative number. We also asked this last question in #25, but we ask it again here because now the child is asked to subtract a negative from a positive number.)

NOTE. c) 6 – -2 = ☐ (**ask all students**). *Look at these two symbols* (point to the subtraction sign and the negative sign). *Do they mean the same thing, or do they mean different things? In what ways do they differ?*

6 + -3 = ☐ One goal for posing this problem is to determine how children make sense of adding a negative number. After each equation ask the following questions:

27. *Do you have a (another) way of thinking about what it means to add -3 to 6?* If the child invokes the commutative property, ask *What if we have to solve the problem the way that it is written on the page (without changing the order). How would you solve it?*
For those 7th and 11th graders who change this to a subtraction problem, be sure to ask something like, *I noticed that when you solved it, you said 6 minus 3, but the problem is written as an addition problem. Why can you make that change?*

28. 3 + ☐ = 0 If the child notes that the problem can be solved *only* with a different operation (e.g., 3 +__= 0 can be solved only if the problem was subtraction, 3 –__ = 0), ask, *If we go back to the way the problem is written on the page, is there anything you can put in the box to make the number sentence true?*

Not possible = NP "I don't know" = ?
Problem not posed = diagonal slash through problem # (or entire page)

PROBLEM IN CONTEXT

29a. *This next question is about a bird flying in the sky and a fish swimming in the water. Imagine that you are standing on the beach. There is a bird flying 20 feet above the water's surface and a fish swimming 5 feet below water's surface.* [Be sure that the picture is showing while you read the text of the problem.]

(1) For 2nd and 4th grade, *Can you explain the situation? What can you tell me about where the bird and the fish are?* Then ask, *How many feet higher is the bird than the fish?*

For 7th and 11th grades, immediately ask, *How many feet higher is the bird than the fish?*

(2) For all grades, if child gives incorrect answer or gets the correct answer for unclear/wrong reasons. *How far apart are the bird and the fish?*

(3) For all grades, if needed. *The fish is friends with the bird and wants to give the bird a high five. The bird doesn't know the fish is coming to visit, so the fish has to go to where the bird is now. How far does the fish have to go to give the bird a high five?*

29b. *Can you write an equation or number sentence that describes this story? Can you explain how this number sentence (equation) relates to the story?*

29c. *Are there other equations that describe the story? If so, please write those equations. (For each equation written) Can you explain how this number sentence (equation) relates to the story?*

If at this point, the child has not referred to the context recently (and has been talking primarily about equations), you may need to ask, *Can you remind me what the story is?* [Feel free to ask this question at any time during this interaction.]

I am going to show you three different equations students have written for this problem. They may or may not match your equations.

**Be sure to explicitly note to the student which question (a, b, or c), you are referring to when you ask whether the equations below describe the story

**First, show the child the same equation that the child wrote for 29b, if the equation the child wrote is one of the equations written below (for most students, the first equation you show will be 29f). Keep the other equations covered. If the child is unable to write an equation, show 29f first.

29d. $20 - -5 = 25$

[If child states that it describes the story because you can do a double switch, ask, *Before you changed it, would this number sentence describe the story?]*

29e. $20 + -5 =$ ___

If the child does not fill in a value, ask*, What number do you think goes in the blank?*

29f. $20 + 5 = 25$

After each equation or expression, ask the following questions:

Would this number sentence (equation) describe this story?

Can you explain how this number sentence (equation) relates to the story?

Not possible = NP "I don't know" = ?
Problem not posed = diagonal slash through problem # (or entire page)

SKIP PROBLEMS ON THIS PAGE IF THE CHILD IS OPERATING SOLELY IN THE DOMAIN OF NATURAL NUMBERS. GO TO COMPARE PROBLEMS (BEGIN @ 36).

30. -5 – -5 = ☐

31. -7 – -9 = ☐ If needed, *Can you explain why your answer wasn't something else, like* _____ (use the opposite of the child's response in the child's language)*?* For example, If the child answers 2, ask why not -2? If the child answers, -2, ask, why not 2? Note that we are trying to get the child to explain how he or she chose the sign of the answer. You could also ask, ***How did you decide whether your answer was negative or positive?*** **(or use the child's language [e.g., *minus* or *regular*]?)**

32. ☐ + -7 = -3

33. ☐ + -2 = -10

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34. 3 – ☐ = -6 **If a 7th- or 11th-grade student** does *not* subtract 3 from both sides, say, *One student solved this problem by subtracting 3 from both sides.* (Rewrite the problem on the paper and write -3 below the 3 and the 6.) *Does that make sense? Can you finish solving the problem using the way she started the problem?*

If the child solves problem by subtracting 3 from both sides (equation code), whether by his or her choice or after our prompting, and then fails to account for the minus sign when isolating the box (in this case, the student is likely to get an answer of -9), point to the minus sign in the equation and ask, *What happened to this* [minus sign] *when you solved the equation?*

35. -2 – 7 = ☐

If the student changes -2 – 7 to -2 + -7 or says something like "They're both negative, so it's -9," be sure to ask about this explicit (or implied) change from subtracting to adding a negative. This is likely to occur only with 7th and 11th graders.

For **everyone** who gives an answer, ask, *Sometimes there is another way to think about a problem. Could you find another way to solve this problem? How else might you solve it?* We want to give students an opportunity to solve the problem in more than one way if they have not already done so. (Moreover, this specific problem is posed so that we can compare the difference, if any, between how the child responds to -2 – 7 = __ vs. -8 – 3 = __).

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36. -8 Point to -8. *Can you read this? What does it mean?*

COMPARISON TASKS (For each, ask how the comparison was made.)

For each pair of numbers circle the larger, write = if they are equal, or write ? if there is not enough information to tell which one is larger. Please read each pair of numbers for me so that when we watch you on the video we know which problem you are solving?

For each problem 37–47, ask, *How did you think about that?*

37. 3 7
38. -7 3
39. -5 -6
40. +20 20
41. -0 0
42. -9 0

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43. - -4 -4
44. – (-4) -4
45. -5 -100
46. ☐ - ☐ If the child has trouble interpreting the ☐, say, *These boxes mean that you can put any number in them as long as you put the same number in both boxes.*
47. *Is there anything you can write in the blank to make the following statement true?*

    5 = -____

For some 4th (and possibly 2nd) graders we may return to problem 46 at the end of the interview and ask the following questions:

If the child answered that the box was larger ask, *Is it larger no matter what number you place in the box?*
Then follow-up with, *What if you put a negative number in the box?*

For 2nd and 4th graders, thank them for sharing their thinking and give them a book to thank the child for thinking so hard during our time together.

Not possible = NP "I don't know" = ?
Problem not posed = diagonal slash through problem # (or entire page)

ALGEBRA TASKS (ONLY for 7th and 11th graders)

For the following, circle the larger, write = if they equal, or write ? if there is not enough information to determine which is larger.

48. x $x + x$
49. x $x + 1$
50. $x + y$ $x - y$
51. $-x$ x
52. 7 x
53. x -7

Refer to problem 51. If response is incorrect, ask, *Is there any value you can use for* x *that would make this (point to –*x *in 51) larger than* x?

54. *If* x < y*, what can you tell me about comparing these two?* [Point to $-x$ and $-y$]? (see student version) If the student solves this problem algebraically, with a rule, or by specific example, ask, *Could you reason about this on a number line?*

ABSOLUTE-VALUE TASKS (Only for Precalculus or Calculus students)

55. *What does* the absolute value of x *mean?*

56. a. *Someone wrote this as the definition of* absolute value. (Hand the student the piece of paper with the definition).

The absolute value of x: $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Can you read this to me (point to definition of *absolute value*)*? What does this mean? Do you think this makes sense for the definition of* absolute value*? Why?*

b. *According to this definition, explain what the absolute value of -2 is.*
Pose this follow-up question, if needed*: I am confused because negative 2 is less than zero. Doesn't this* (circling the $-x$) *mean that my answer should be negative?*

End of the Interview. Thank the student for taking the time to thoughtfully answer the questions. Give the student a gift card and have the student sign the receipt to indicate that he or she received the gift card.