WHAT SENSE DO CHILDREN MAKE OF NEGATIVE DOLLARS?

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About the Project

- Mapping Developmental Trajectories of Students' Conceptions of Integers
  - aka Project **Z**
- 160 interviews in spring of 2011
  - 40 children each in grades 2, 4, 7, and 11
  - Many common tasks across grades
- Focus today
  - Grade 7 data
  - One of many tasks

Contexts for Integers

- Review of 5th- and 6th-grade textbooks revealed that 94% used stories involving money in integer instruction (Whitacre et al., 2011)

Money Problem

- *Yesterday, you borrowed $8 from a friend to buy a school t-shirt. Today, you borrow another $5 from the same friend to buy lunch. What's the situation now?*
  - *Do you owe your friend money?*
  - *Does your friend owe you money?*
  - *How much money?*

- How might students answer?

Money Problem

- *Write an equation or number sentence to describe this story problem (and answer).*

- What equations might students write?

Money Problem

- *Here are three equations that other students have written to describe this story. Do these make sense to you? Why or why not?*
  i. $-8 + -5 = -13$
  ii. $-8 - 5 = -13$
  iii. $8 + 5 = 13$

- How might students respond?

Two Seventh-Graders' Responses

- Evelyn wrote 8 + 5 = 13.
  - Clip. Her reaction to -8 + -5 = -13
- Carla wrote -8 + -5 = -13.
  - Clip. She also wrote and explained 8 + 5 = 13.

Evelyn's Thinking

Carla's Thinking

Regular Numbers

- What are *regular numbers*?
- Regular numbers are neither positive nor negative.
- Integers convey
  - Direction,
  - Magnitude.
- Regular numbers convey
  - Magnitude.

Methods

- Grade 7 data
- Open coding of responses led to a distinction of interest:
  - *Perspective* (3 codes)
- Codes were refined and used to code the whole data set.
- We assessed inter-rater reliability on 25% of the data.
  - Coders agreed on 90% of coding decisions.
- Today
  - We offer examples of each perspective.
  - We also report frequencies of each.

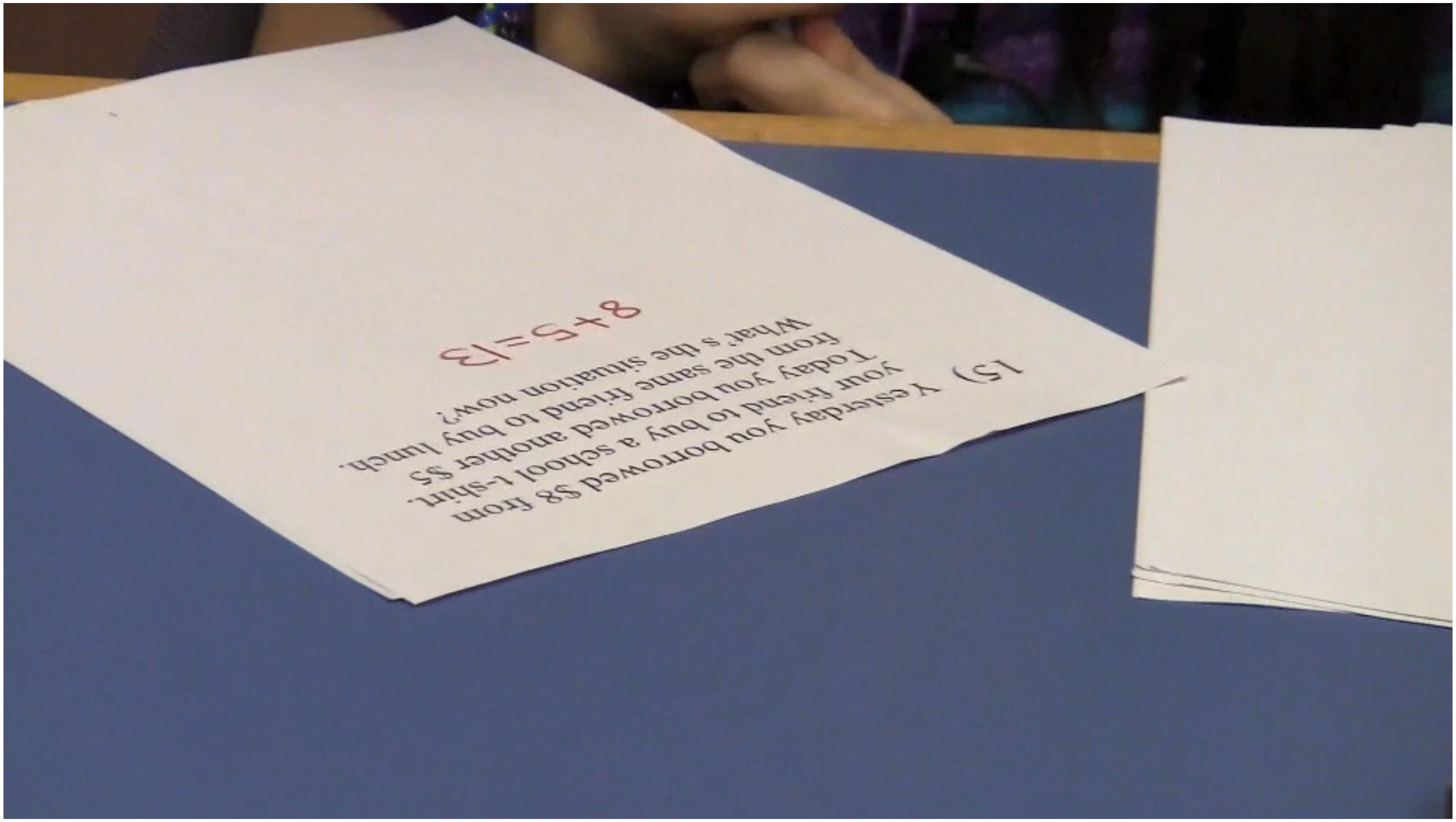
Perspective?

- First distinction
  - Using *regular numbers*, as opposed to integers
  - *Perspectiveless*
- Second distinction
  - How are integers related to the money context?
  - *Conventional* and *Unconventional Perspectives*

Conventional & Unconventional Perspectives

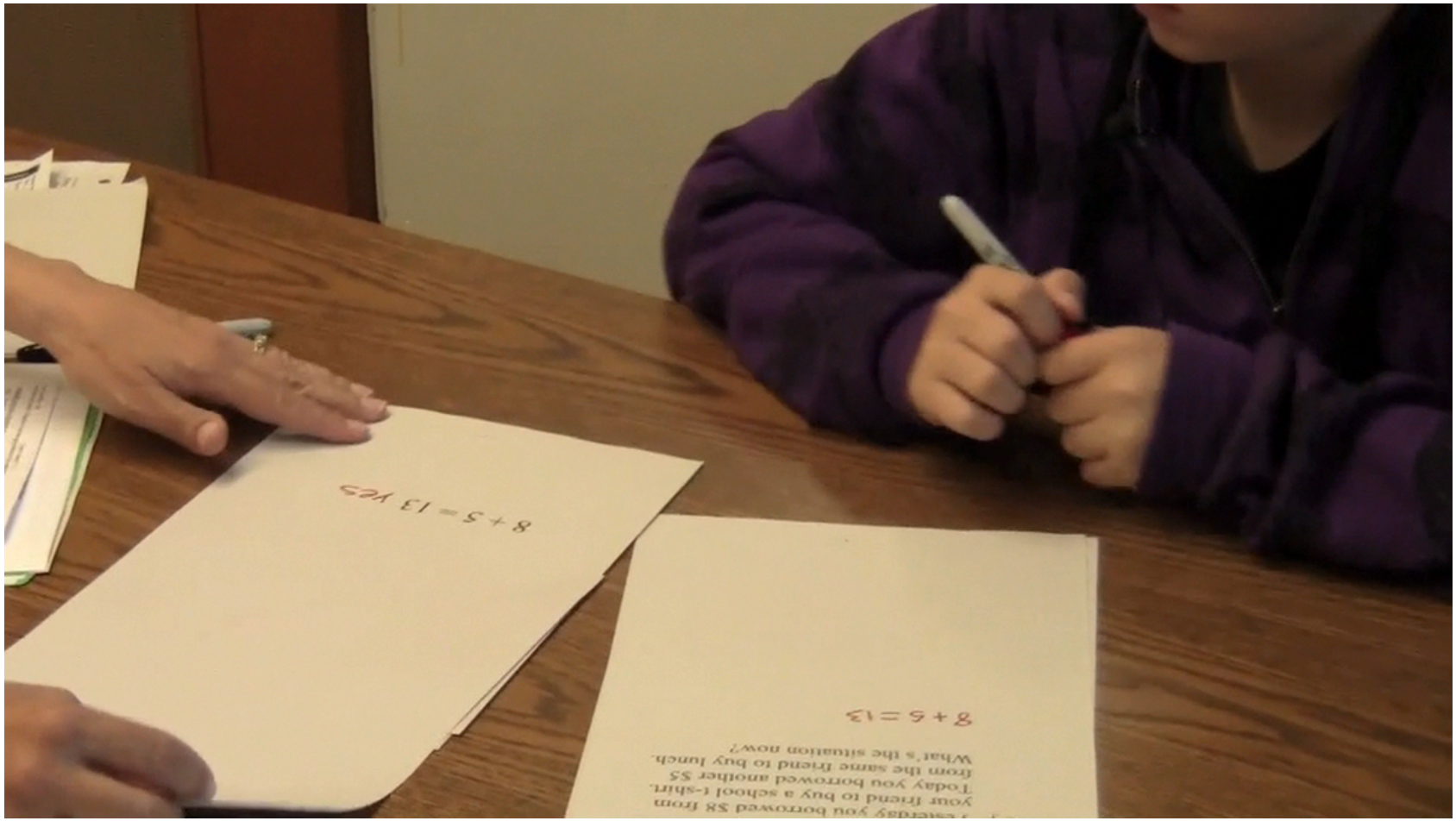
- Integers convey directional information:
  - Who owes money to whom?
- *Conventional*
  - Negatives denote money owed
  - Positives describe the lender's situation
- *Unconventional*
  - Negatives denote money lost (loaned).
  - Positives denote money gained (borrowed).

Perspectiveless



Conventional Perspective

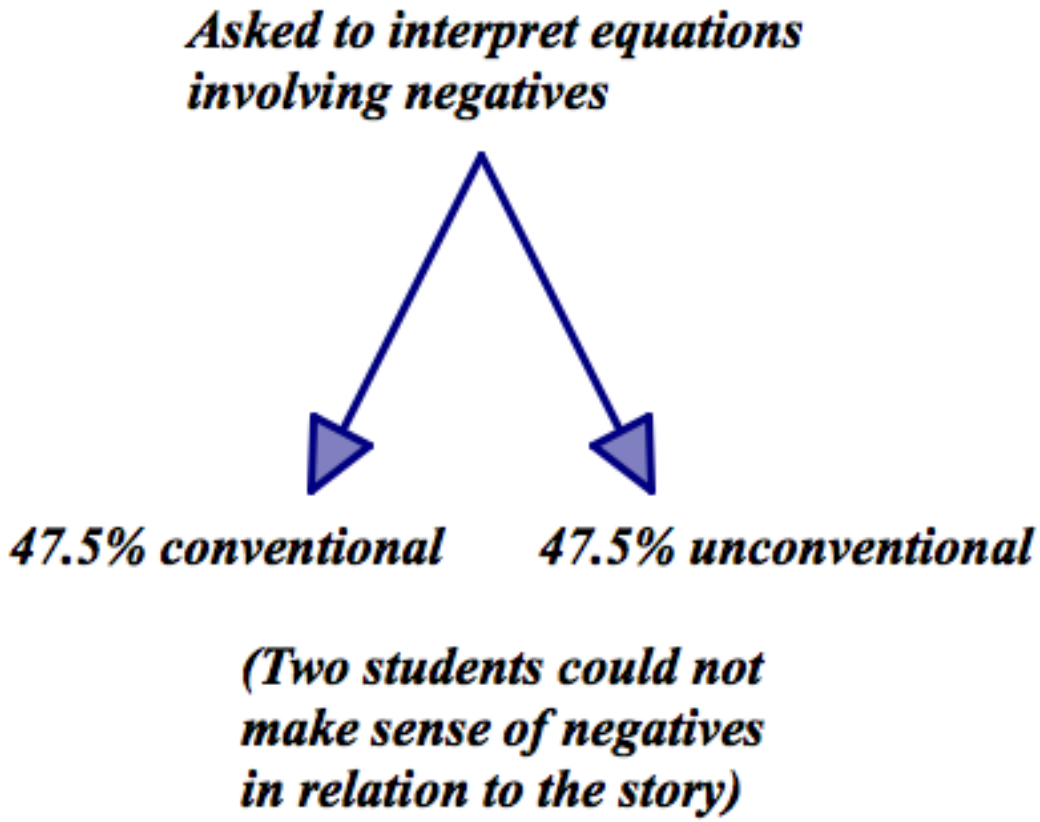
Unconventional Perspective



Results

- How many 7^{th} graders correctly solved the money problem?
  - 100%
- How many wrote an equation involving negatives?
  - 20%
  - Two of these also wrote $8 + 5 = 13$.
- 77.5% did not use negatives.
  - One other
- How did they interpret equations involving negatives?

Results


Asked to interpret equations involving negatives
47.5% conventional
47.5% unconventional
(Two students could not make sense of negatives in relation to the story)

Summary of Results

- 100% of 7th graders correctly solved the problem!
- Only 20% invoked negative numbers.
- When explaining/interpreting equations involving negatives in relation to the money problem,
  - Half used a conventional perspective;
  - Half used an unconventional perspective.

Conclusions

- All students solved the problem correctly.
- Many did not think about it in terms of integers.
- Regular numbers are part of students' worlds.
- Almost all students were able to interpret equations involving negatives in relation to the money problem.
- Half of 7^{th} graders interpreted these unconventionally.
- (Unconventional perspective was less common among 11^{th-} grade pre-calc students).

Implications

- Sensitivity to the distinction between regular numbers and positive numbers
  - Learning to inhabit various mathematical worlds
- The relationship between integers and contexts is not trivial.
  - Negatives can be used to represent owing.
  - But how?
  - To what end?

Discussion

- We want to know what you think about these ideas and these results.