Children's Negative-Number Obstacles and Similarities to Mathematicians
(Philipp & Bishop, TDG, February 14, 2014)

Obstacle	Children's responses	Mathematicians' related responses
The existence of quantities less than nothing and the lack of a tangible, concrete, or realistic interpretation for negative numbers (*Negatives rejected*)	"Negative numbers aren't really numbers because we don't really count with them in school. And there's no negative 1 cube (holds up a unifix cube)." A number is "how you know how much something is." (Violet, second grade) "A number shows how much of something you have; … [-8] is not actually a number because it's less than a number. … It just doesn't really have any volume for it, like what it has." (Elena, third grade) "Zero is nothing and negative is more nothing." (Rebecca, second grade)	The Indian mathematician Bháscara I explained, "People do not approve of a negative absolute number;" thus, negative solutions were considered "incongruous" (Colebrook, 1817, pp. 216–217). Fibonacci and Descartes did not accept negative solutions unless the result could be interpreted as something positive. "Above all, he [the student] must reject the definition still sometimes given of the quantity -*a*, that it is less than nothing. It is astonishing that the human intellect should ever have tolerated such an absurdity as the idea of a quantity less than nothing; above all, that the notion should have outlived the belief in judicial astrology and the existence of witches, either of which is ten thousand times more possible." (De Morgan, 1902, p. 72)
Removing something from nothing or removing more than one has (*Subtrahend < Minuend*)	"Three minus five is zero because you have 3 and you can't take away 5, so take away the 3 and it leaves you with zero." (Sam, first grade) When asked to solve 3 – 4 and 3 – 3, Sam answered 0 to both. "Three minus five doesn't make sense because three is less than five." (Nola, first grade) When solving 3 – 5 = □, Andrew (second grade) replied, "How come there's 3 and take away 5? I don't have enough. 'Cuz look there's 3 (hold up 3 fingers) and I cannot take away 5 'cuz there's not enough."	"I know some who cannot understand that to take four from nothing leaves nothing" (Pascal, 1669/1941, p. 25). "3 – 8 is an impossibility; it requires you to take from 3 more than there is in 3, which is absurd. If such an expression as 3 – 8 should be the answer to a problem, it would denote either that there was some absurdity inherent in the problem itself, or in the manner of putting it into an equation." (De Morgan, 1902, p. 104)
Counterintuitive situations involving routine interpreta-tions of addition and subtraction (*Addition cannot make smaller*; *Subtraction cannot make larger*)	"4 + □ = 3 is not a real problem. It's not true" [he crossed out the problem]. "Four minus 1 would equal 3." (Brad, first grade) In response to the problem 6 + □ = 4, Brian (first grade) said, "What's that plus for? Isn't it supposed to be a minus?" In response to the problem 5 – □ = 8, Ryan (first grade) said, "I wouldn't be able to do it because it would always be behind 8 if it was minus something. Because if it was minus 0 it would be 5. It [the difference] would always be behind 8."	Diophantus claimed that the equation $4x + 20 = 4$ was "absurd" because the 4 was less than the 20 units that were added (Heath, 1964, p. 200). D'Alembert (1751/2011) argued that the equation $x + 100 = 50$ should have involved subtraction instead of addition and been written as $100 - x = 50$.