A CGI Approach to Integers: Helping Teachers Structure Their Intuitive Knowledge About Children's Understandings of Negative Numbers

Ways of Reasoning About Integer Arithmetic

On the basis of a large-scale study of 160 K–12 students' integer-problem solving, we have identified five broad categories of reasoning that students use when solving open number sentences (decontextualized problems such as -3 + $\square$ = 6). Definitions of the *Ways of Reasoning* are below. Additionally, we include the patterns of use and frequency with which different ways of reasoning are used across grade levels.

Ways of Reasoning	Definition	Ways of Reasoning % Use (by total # of problems)			
		2/4 No[a]	*2/4 Yes*	*7th*	*11th*
Order	Using the sequential and ordered nature of numbers to reason about a problem (e.g., counting strategies or a number line with motion).	0%	33%	38%	19%
Analogical	Relating negative numbers to another idea/concept and reasoning about negative numbers on the basis of behaviors observed in this other concept. Negative numbers may be related to a countable amount or quantity and tied to ideas about cardinality/magnitude, or they may be related to contexts.	0%	11%	20%	16%
Formal	Treating negative numbers as formal objects that exist in a system and are subject to fundamental mathematical principles that govern behavior in that system. Formal strategies often involve comparisons to other, known, problems so that the logic of the approach remains consistent and underlying structural principles are not violated.	0%	3%	12%	24%
Computational	Using a procedure, rule, or calculation to arrive at an answer.	13%	13%	53%	75%
Alternative	Using strategies that reflect incomplete or limited views of negative numbers and may have invalid mathematical foundations. At times, the domain of possible solutions is locally restricted to **W**.	93%	59%	14%	<1%

Note. Because students can use more than one way of reasoning to solve a problem, column-percentage sums are larger than 100%.
[a]Students without negative numbers in their number domains.

- Young children can reason productively about negative numbers! Half of all 2nd and 4th graders in our study had heard of negative integers *before* any school-based integer instruction, and three fourths of 2nd and 4th graders solved at least 1 integer problem correctly.
- *Flexibility* is a measure of the variety of ways of reasoning (WoR) used to solve integer tasks; it indicates whether a student uses primarily one WoR or chooses different WoR depending on the affordances of the problem. We found that flexibility increases when we move up grade levels and is positively correlated with performance in our data, both across grades (r = .6) and within grades (r = .36, .40, .34 for 2/4, 7th, and 11th, respectively).

Visit the Project Z Website at http://www.sci.sdsu.edu/CRMSE/projectz/index.html

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Randolph Philipp (rphilipp@mail.sdsu.edu), San Diego State University & Jessica Pierson Bishop (jpbishop@uga.edu), University of Georgia

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