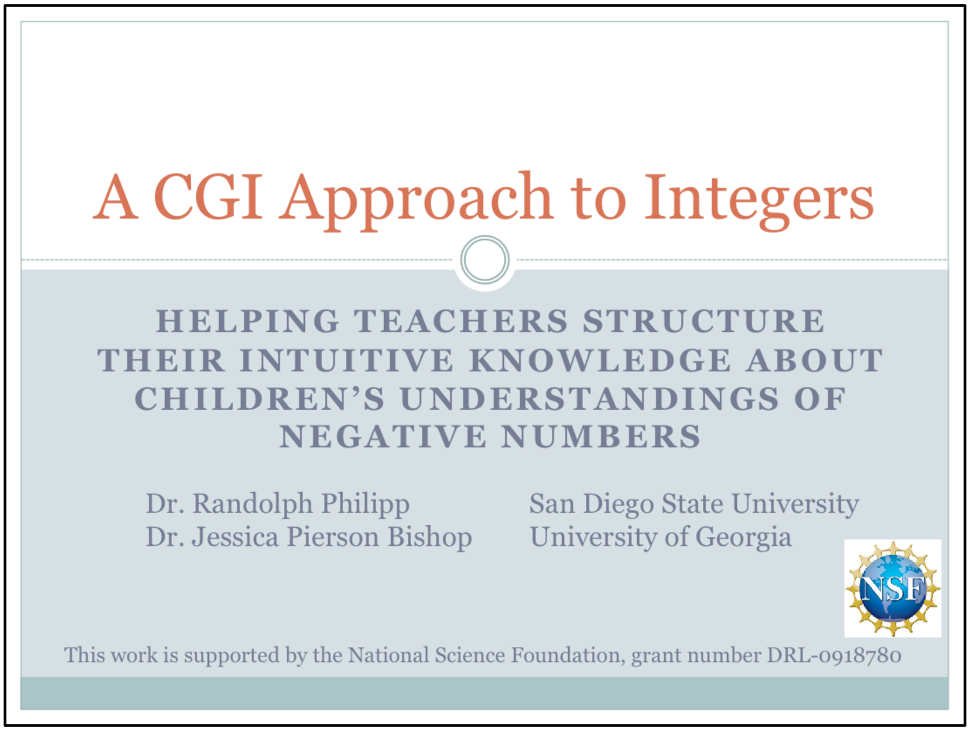


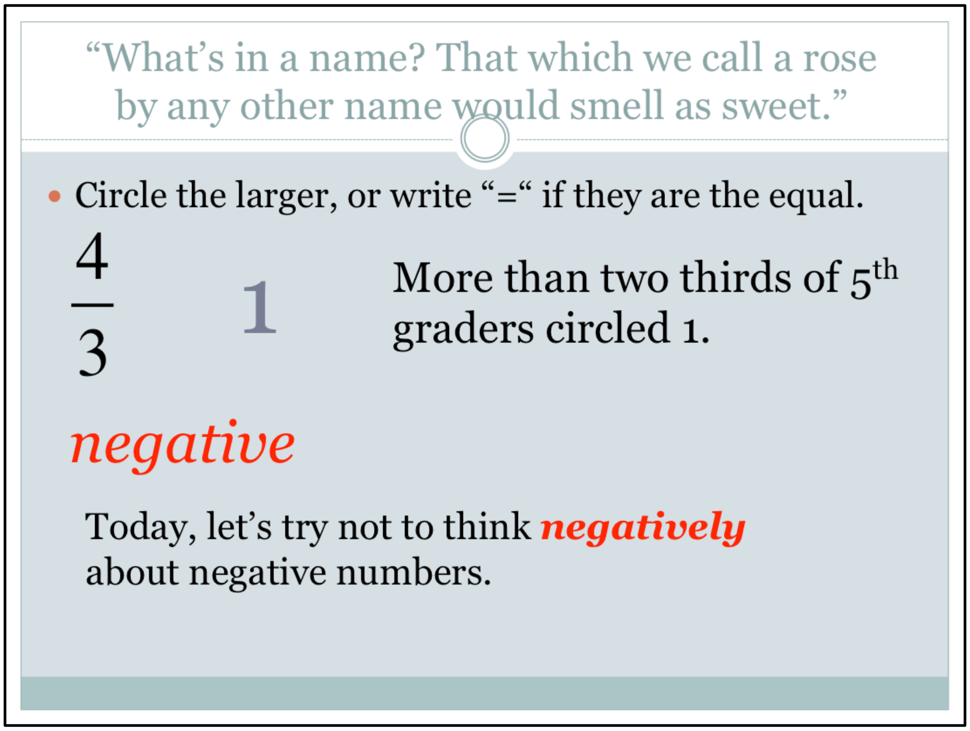
Thank you, Linda (Foreman), for the gracious introduction.

Jessica and I are delighted to be with you today.

Let's Start with a Game

- Write a number. Any number. Circle it.
- Add 10
- Multiply by 2.
- Add 4.
- Add 8.
- Divide by 2.
- Subtract 17.
- Subtract your original circled number.
- Draw a rectangle around your final number.

We are just back from lunch, maybe a little mathematics after lunch can help with digestion So we are going to begin with a little game. You'll need a sheet of paper for this game.
Write a number—any number you want, but you'll be operating on the number, so consider that.
Circle the number. We'll refer to this as your *circled number*. (n)
Add 10. Write your new sum. (n + 10)
Multiply by 2. Write your new product. (2n + 20)
Add 4. Write what you have now. (2n + 24)
Add 8. (2n + 32).
Divide by 2. (n + 16)
We've almost finished the game. Subtract 17. Feel free to use any available technology, including your neighbor's fingers or toes. (n – 1)
And now, I want you to subtract the circled number from the number you selected at the beginning of the game.
This is your *final number*. Make a rectangle around your final number.
Now I am going to see if I can read your minds. Everyone concentrate on the number in the rectangle. Oh my, your vibrations are coming through strongly. I see many numbers, but one seems to be looming. Raise your hand if the final number in your rectangle is negative 1!
Wow, look at that. Clearly, this crowd has negatives on their mind. So let's talk about negative numbers today!
(By the way, if you want to know how this works, talk to Jessica or me later.)
0–2



We will take a few lines from Romeo and Juliet by William Shakespeare, who, if he were still alive, would turn 450 years old this April.

Turns out that names are important.

For example, we gave some comparison tasks like this to fifth graders, after they had learned fractions for several years.

I bet you know why. They said that fractions are less than 1. Where did they learn that fractions are less than 1?

From school. From life. Consider the following: "I told you to clean up your room, and you have cleaned up only a fraction of this mess! No parties for you until this room is clean!"

How we think about the name of something is important.

Today, we are talking about numbers that are *negative.*

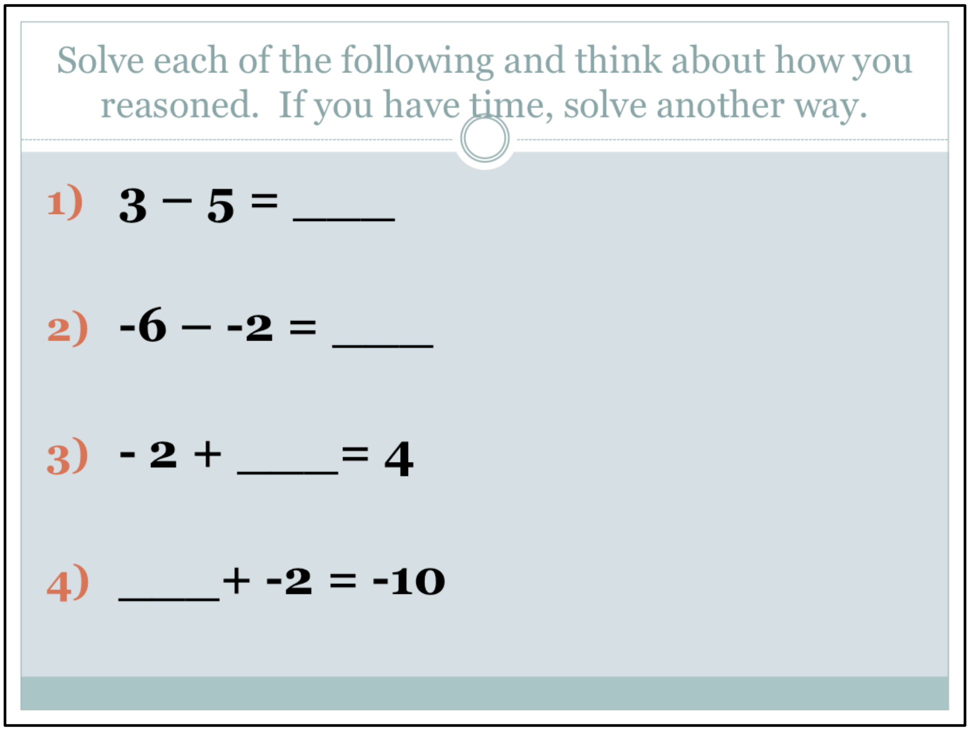
How do we use the term *negative* in everyday usage?

Today, let's agree not to think negatively about negative numbers. I guess we could say that we should think positively about negative numbers, but that might be too confusing. And actually, later, we'll see that thinking positively about negative numbers is often what happens!

So the next time you throw a number party, why don't you consider inviting more than just positive natural numbers. Have you seen 9/7 dance? Wow, 9/7 has some moves! And -5 can tell some wonderful jokes! And if the numbers you are inviting all seem to be behaving a little too rationally for your taste, invite pi! I admit, pi's stories seem to go on and on and never end, but without pi, the only dancing we get to do is square dancing, and, hey , there is nothing wrong with square dancing, but I'm just saying, "Shake things up a bit!"

2–5.

So let's shake things up a little!



We would like you to solve each of these four open number sentences and take note of how you were reasoning.

Ask people to share their reasoning. We write their reasoning in a Word file.

5–8 minutes to think
8–11 minutes to share

(Below is what they shared and what we wrote for them to see.)
#1
- Start at 3 on # line, move backward 5.
- Number line, distance between 3 and 5, and in which direction
- I broke -5 into -3 and -2, and I know 3 + -3, or 3 minus 3…

#2
- I owe $6, and someone took away $2 I owe, so I owe, only $4.
- I have six negative chips, and I took two negative chips away, and I have four negative chips left.

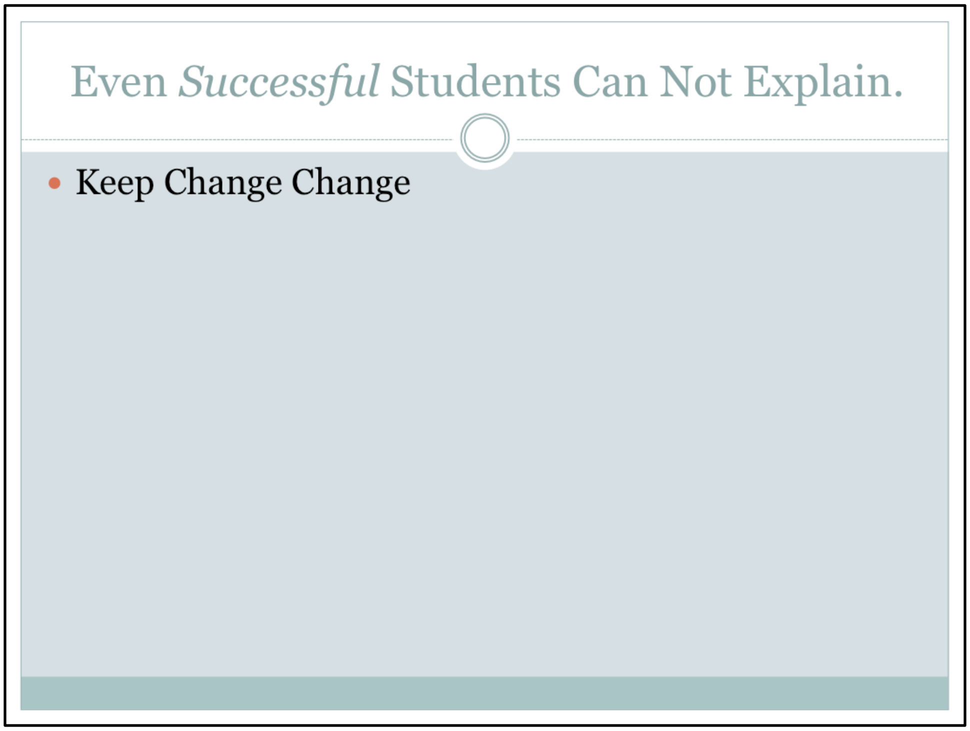
Our presentation today is based on work funded by the National Science Foundation and is part of a larger study in which we piloted more than 90 interviews while developing the interview tasks, videotaped 160 interviews (lasting about 1.5 hours each) of students in grades 2, 4, 7, and 11, and analyzed those assessments. We are currently in the 5th year of a 3-year project.

11–11:30

So, Why Negative Numbers?

- Even secondary-school students who can successfully operate with negatives have trouble explaining.

11:30–12



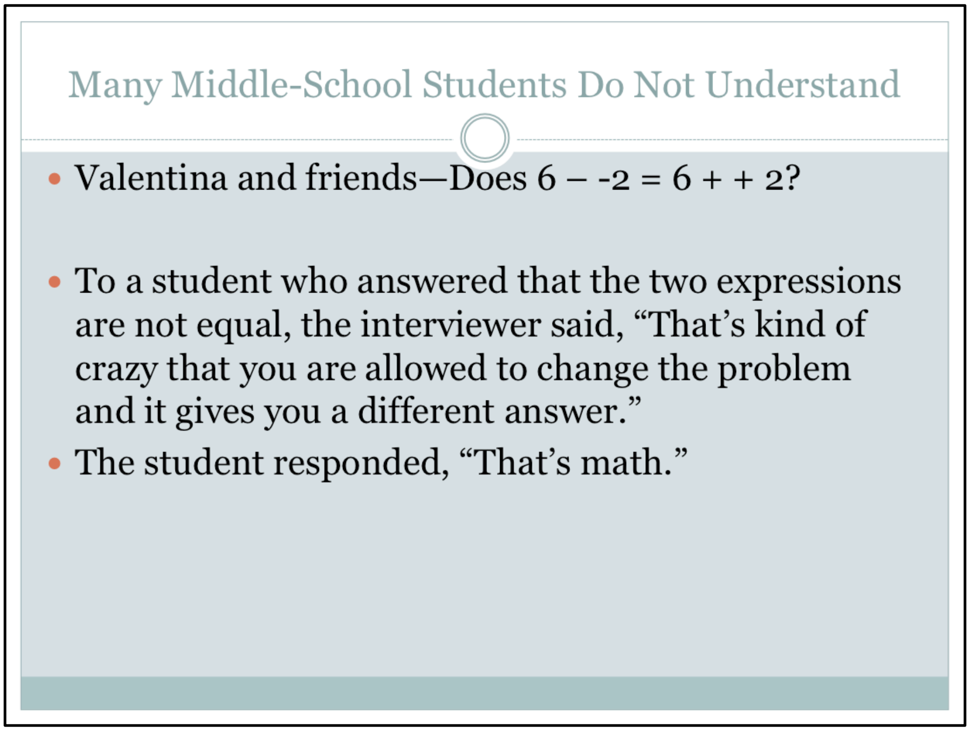
12–14:30 video
This morning, Rebekah Elliot and Wendy Rose Aaron presented a fascinating session on practice. A subgroup I was in discussed what *orientations to the discipline* means. I think that these clips display *an* orientation to the discipline whereby mathematics often is about doing something without understanding why it works.

14:30–15 By the way, we find noteworthy that these are among the most successful students, coming from 11th-grade precalculus or calculus classes.

So, Why Negative Numbers?

- Even secondary-school students who can successfully operate with negatives have trouble explaining.
- Many middle-school students do not understand what they are doing with negatives.

15–15:30



When asked to evaluate 6 – -2, many 7th graders evaluated the expression by changing both signs to positives, adding, and then responding, “Eight.”
We asked them whether the answer, before they changed the signs, was *8*.

15:30–16
16–19:30 Video

I want to make sure that you heard the end of the clip. The interviewer said, “That’s kind of crazy that you are allowed to change the problem and it gives you a different answer.”
The student responded, “That’s math.”

The Common Core State Standards (CCSS) of Mathematical Practice

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision (in language and mathematics)

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

19:30–19:40

The CCSS of Mathematical Practice

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7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

19:40–20:10

What Is Mathematics to Students?

70	76
- 23	- 23
53	53

"Yes, math is like that sometimes."

20:10–21:30

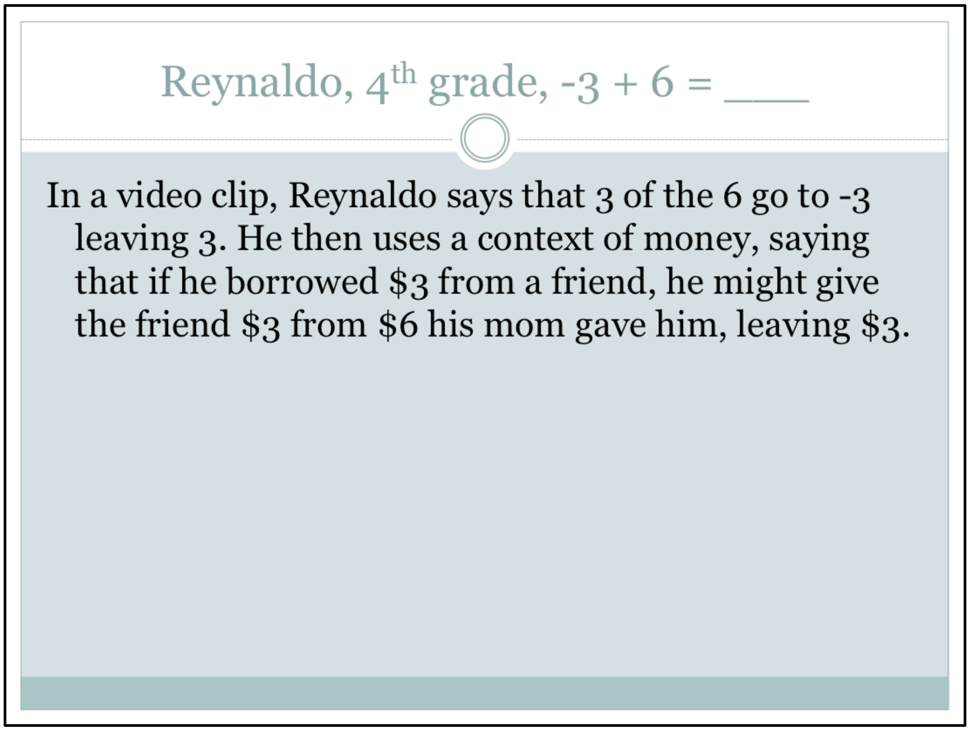
Confused reasoning does not apply only to integers.

(Randy shared an anecdote about an elementary mathematics methods course he taught and his students' interviews of 2nd graders. In preparation for the interview, he told the students that a common incorrect answer to 70 – 23 is 53, and he suggested that if it arose, the student present the problem on the right as a means to induce disequilibrium. One student reported her experience: She pointed out to the second grader, "Well, that's interesting. Here you have ***70*** minus 23 and here you have ***76*** minus 23, and in both cases you got the same answer of 53." The child responded, "Yes, math is like that sometimes."

So, Why Negative Numbers?

- Even secondary-school students who can successfully operate with negatives have trouble explaining.
- Many middle-school students do not understand what they are doing with negatives.
- Many young children hold informal knowledge about negatives on which instruction might be based.

21:30–22

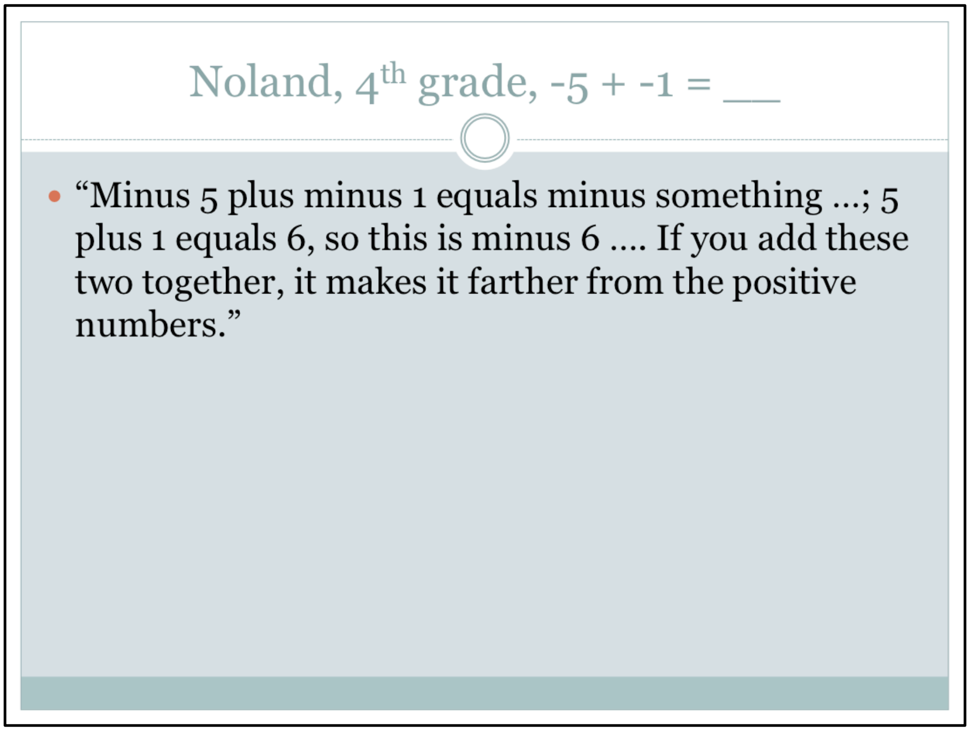


Video of Reynaldo solving -3 + 6 = ____, using debt

22–23:30 Video in which Reynaldo says that 3 of the 6 go to -3 leaving 3. He then uses a context of money, saying that if he borrowed $3 from a friend, he might give the friend $3 from $6 his mom gave him, leaving $3.

Jessica

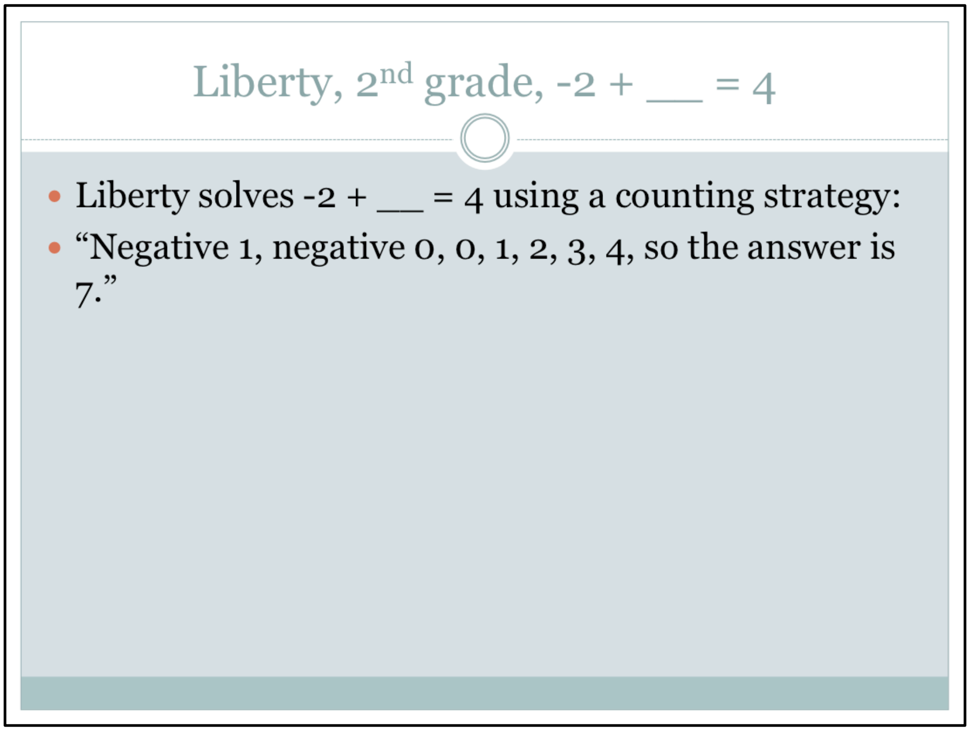
23:30–24 Jessica (after giving the audience a moment to turn to a neighbor and summarize Reynaldo's reasoning), to help the audience make sense of Reynaldo's reasoning, noted the use of *owing* as a useful interpretation of negative numbers and mentioned the implied use of additive inverses.



Noland solved -5 + -1 = __ using Negatives like Positives
24 – 24:50 video

(The audience was given a moment to turn to a neighbor and summarize Noland's reasoning.)

24:50–25:30 Jessica clarifies Noland's reasoning by mentioning his comparison of negative numbers to regular numbers and highlighted Nolands' implied use of order (based on his statement of "moving farther from the positives").



Liberty solving -3 + 6 = __ using a counting strategy.

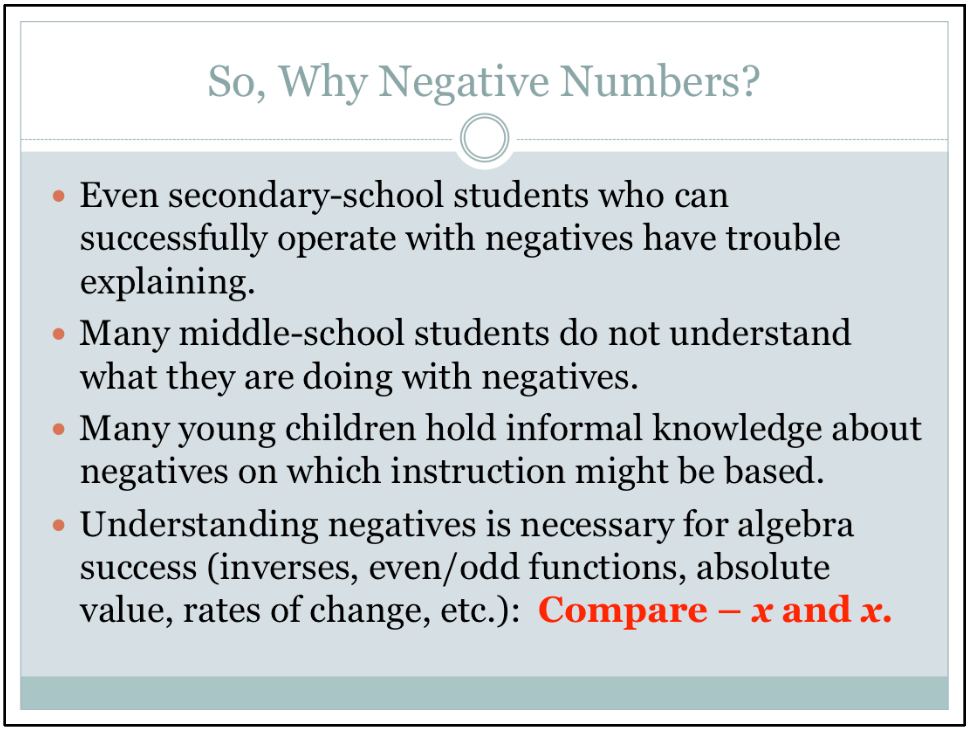
Jessica asked the audience to turn to a neighbor and predict what Liberty might do to solve this.

After they thought, Randy took out a $20 bill and said, "I will give this to anyone who correctly predicts Liberty's reasoning."

25:30–26:20 Video
"Negative 1, negative 0, 0, 1, 2, 3, 4, so the answer is 7."

(No one claims the $20. Randy expresses relief and puts the money away.)

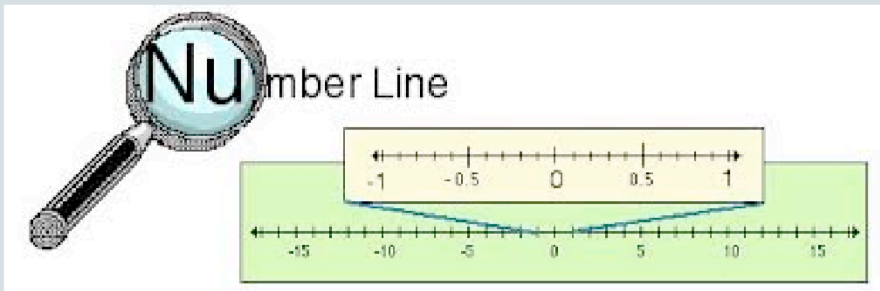
26:20 – 27 27 Jessica mentions the extension of counting strategies into negative numbers and the symmetry of numbers suggestive of a worthy question—is there any such thing as a Negative 0?
We ask the audience to vote on this. (There is a split vote, with some audience members abstaining.)



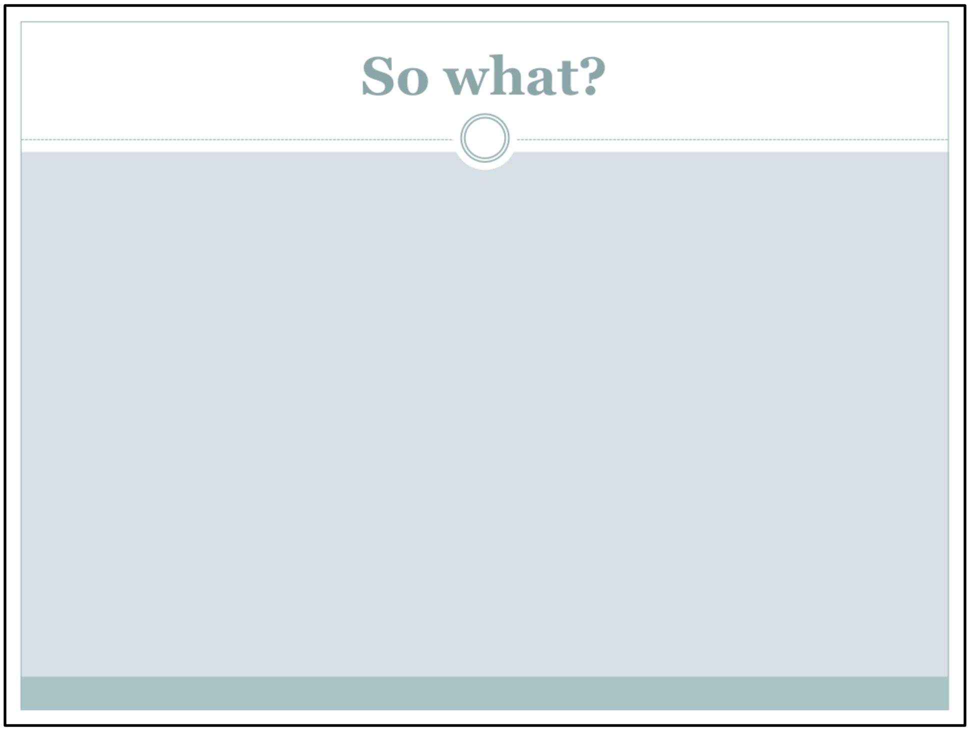
27 – 27:30
Noted that the task in red is challenging. This task involves thinking about negatives, but it also involves thinking about variables.
By the way, we will not have time to discuss this comparison today, but we have found that the way one thinks about *dash x* affects the way one views this. Three ways people refer to this are as *minus x, negative x,* and *the opposite of x.*

One Last Reason…

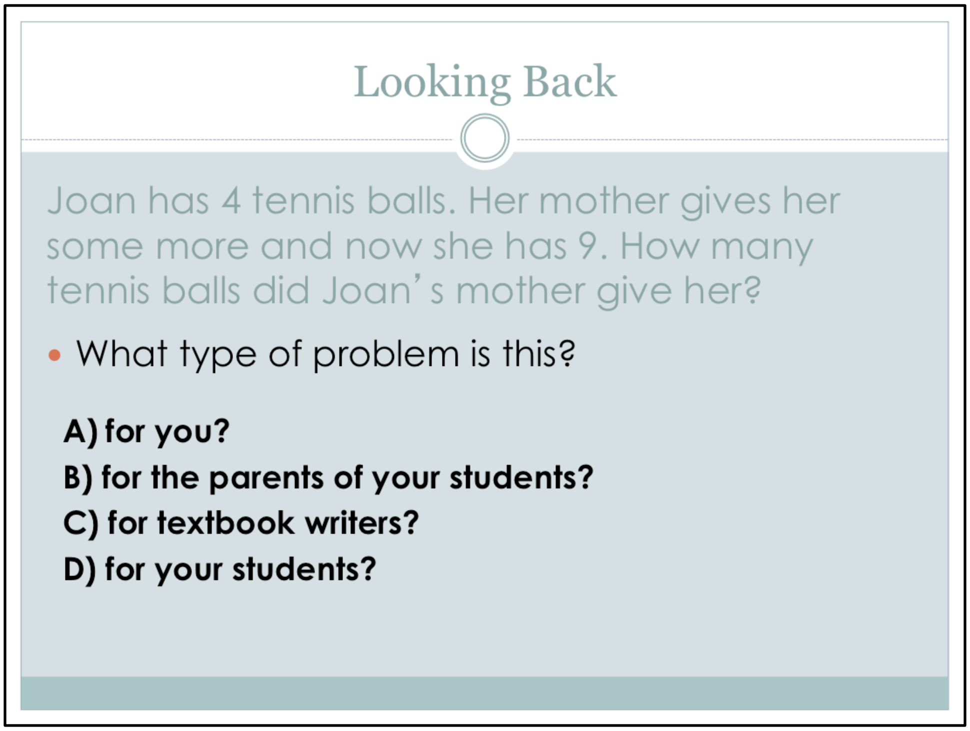
- Negative numbers comprise (almost) half of the reals!



27:30–28



Jessica, why did you slip this slide in here? You know that the last thing we want people in the audience asking is this!



When we can tap into children's informal reasoning, we should do so because then we build on their understanding. Collectively, researchers have successfully applied this approach in a variety of content areas. For example, in the area of whole number addition and subtraction, consider this problem. What kind of problem is it?

Turns out that the answers to these four question are likely not the same. Furthermore, in the past 30–40 years researchers have become very interested in the answer to question D, whereas in the past we might have been interested only in how mathematicians thought about this.

28–28:30

CGI Join and Separate Problems

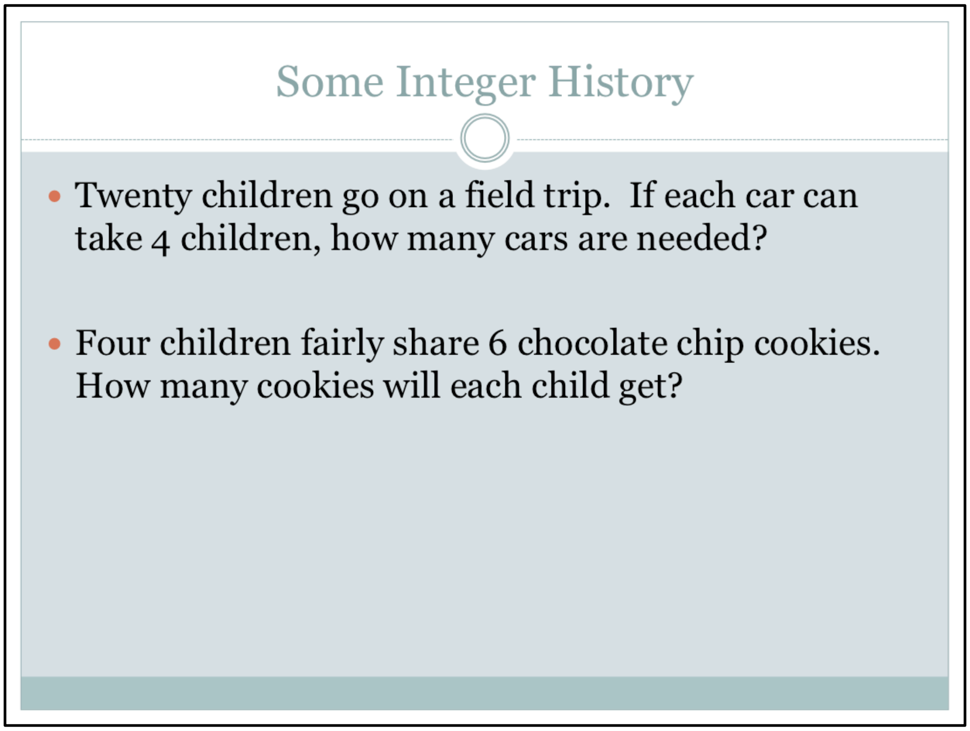
From Carpenter, Fennema, Franke, Levi, and Empson, 1999

	Result unknown	Change Unknown	Start Unknown
Join	Leon has 9 pencils. He gets 5 more pencils. How many pencils does Leon have now?	Joan has 4 tennis balls. Her mother gives her some more and now she has 9. How many tennis balls did Joan's mother give her?	Mary has some marbles. She went to a party and won 5 more, and now she has 11. How many marbles did she have before the party?
Separate	Lesley baked 9 cookies. Kevin ate 3 of them. How many cookies are left?	Eleven children were playing in the sandbox. Some children went home. There were 3 children left still playing in the sandbox. How many children went home?	Mary had some bookmarks. She gave 3 to Kevin. Now she has 9 left. How many bookmarks did she have to start with?

During the lunchtime plenary address yesterday, Linda Levi addressed some of the work done in whole number addition, subtraction, multiplication, and division. For example, we know that children tend to focus on the action and the location of the unknown, and so whereas adults might think of this problem as subtraction, children tend to think of this problem as a join-change-unknown problem, otherwise known as a missing-addend problem.

This kind of knowledge is useful to teachers who, on the basis of this understanding, actually notice their students' thinking in more sophisticated ways, making it possible for them to respond to students, in the moment, in ways that support students' richer and more sophisticated learning and enact more of the mathematical teaching practices that Peg Smith helped us think about in her opening keynote presentation on Wednesday evening.

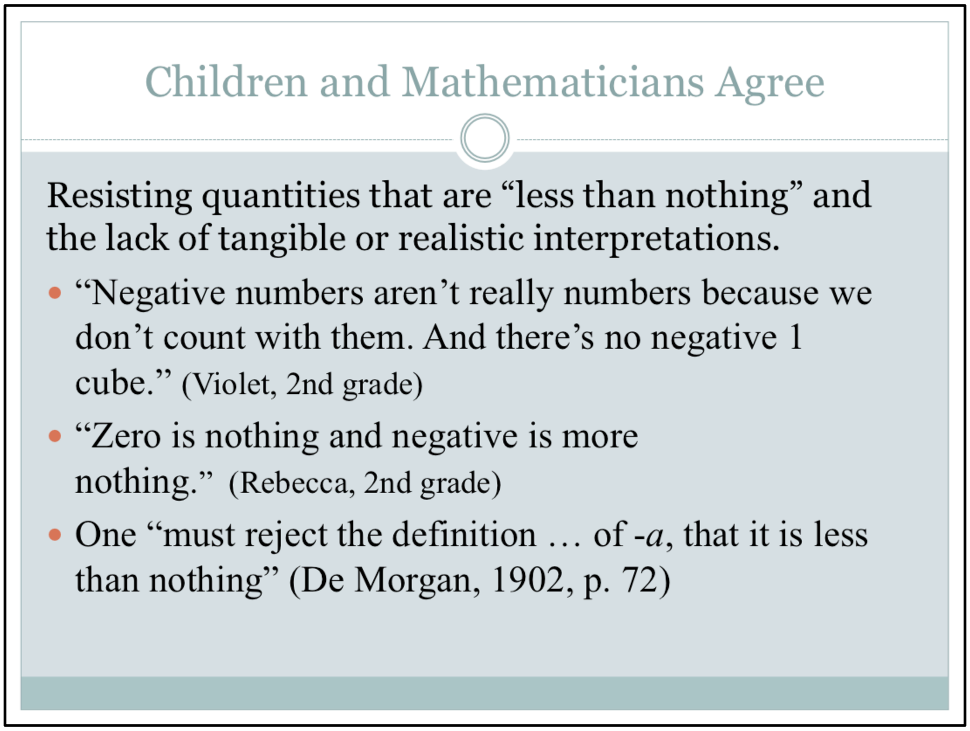
28:30≠29:30



Children, even before formal instruction in whole number division or fractions, are able to make sense of and grapple with problems such as these, thereby opening the door for students to think about division and about fractions as early as first grade or even Kinder. Even more important, when students are invited to think about such problems before they are formally taught explicit procedures, they experience mathematics as a creative, sense-making, activity.

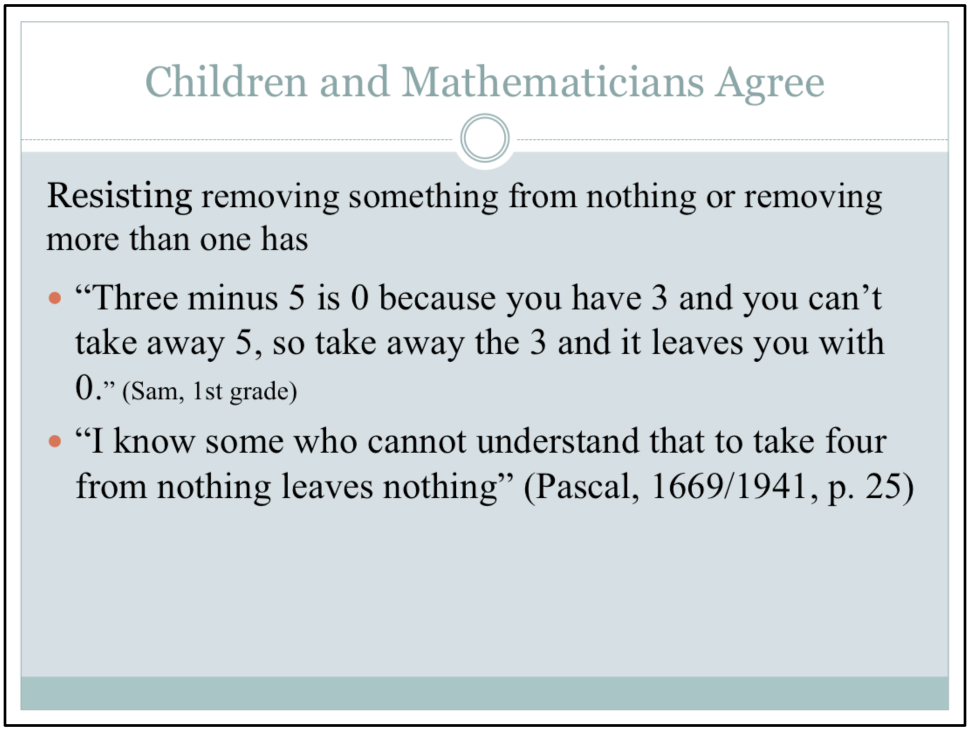
These problems involve context, and when we set out to think about integers, we looked at contexts. But interestingly, we found that when we gave students contexts, such as owing money or increasing or decreasing elevation, they generally avoided using negative numbers. I can talk about a debt as negative dollars or a loss of yards in football as negative yards, but when was the last time you watched a football game and someone said, "Wow, that guy just gained negative 3 yards?."

29:30–30:30



This is not at all surprising, though. Historically, mathematicians resisted negatives for many centuries.

30:30–31:45



31:45–32:15

And my personal favorite example, from the comedian Louis C.K., who said,

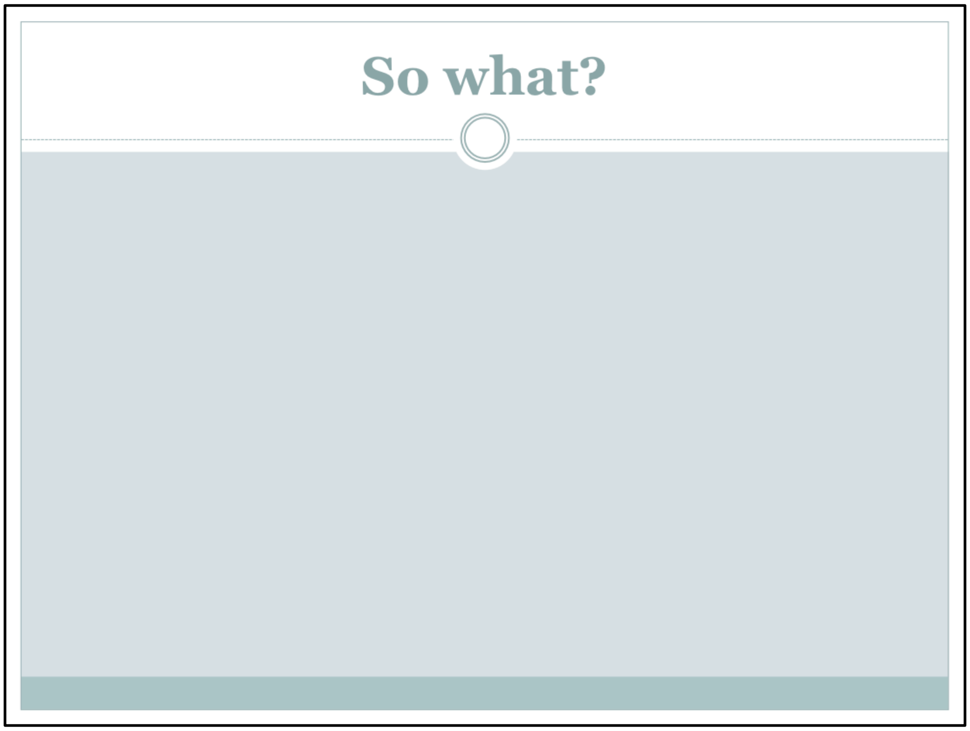
"You ever have negative money? That's depressing, isn't it? You look in your bank account: Negative ten dollars. That's how much I have now. Negative ten. That means I don't even have no *money now! I wish I did. I wish I didn't have anything. I wish I just had nothing, but I have less than that. I don't have* none. … *Someone could come up to me (and say), 'Take this. It's free.'… [I'd have to say] That costs nothing! I can't afford that. That's more than I have. I gotta raise ten bucks to be broke."*

This is funny precisely because it makes so much sense, and yet, at the same time, it is ridiculous. Kind of like negative numbers!

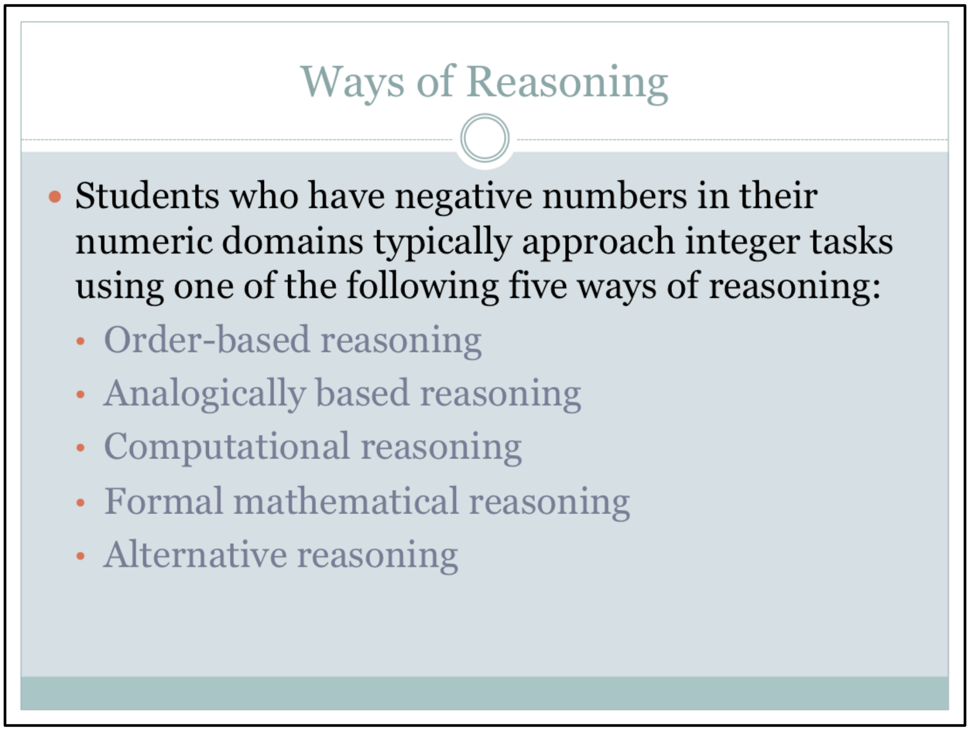
Ian Whitacre, a colleague on our integer project, once had a girlfriend who suggested to him, while at the check-out line of the market, that we should have negative coins so that if something cost $3.95, we could give $4 and a negative 5 cent coin. But then she realized that if we had negative coins, we'd all conveniently lose them. Ian, of course, being no dummy, did not invent negative coins, but he did marry that woman! And they are positively happy!!

We love contexts and encourage their use. But when we talked with our advisory board, including Tom Carpenter, one of the founders of CGI, they encouraged us to go with what might be useful for children, and so open number sentences played a major role in our research.

Now, let me turn this over to Jessica, who will introduce us to ways of reasoning about negative numbers.



Ok. Maybe I'll let Jessica answer this question. Jessica?



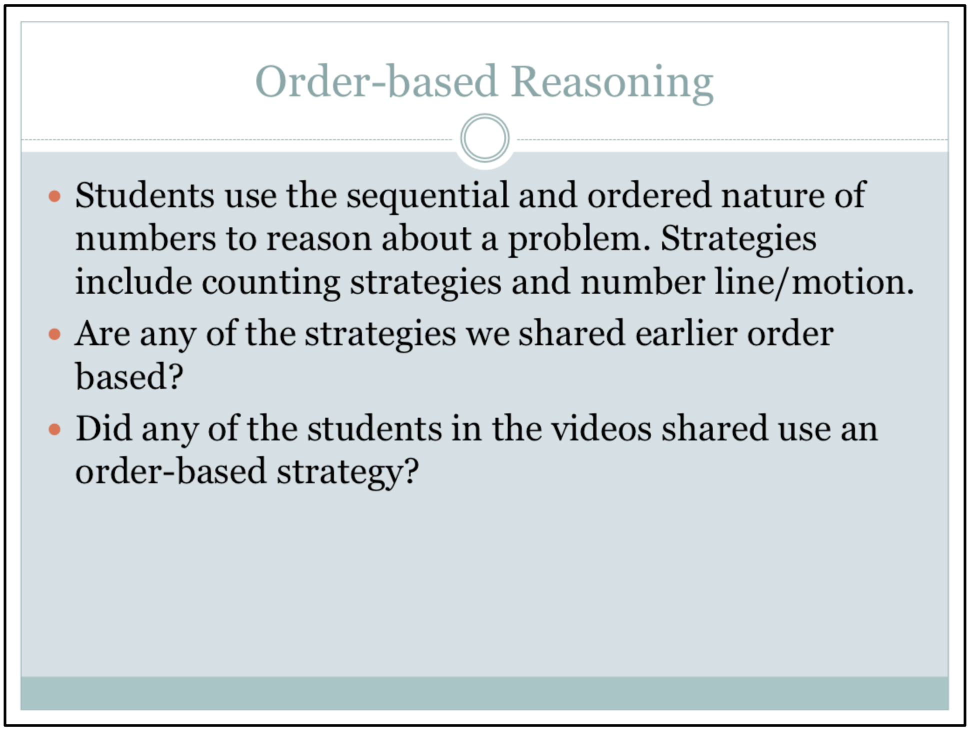
Ways of Reasoning [2 minutes]

So what have we learned in the last 5 years? A lot!

In our project we have identified five Ways of Reasoning with respect to integers and integer addition and subtraction. We identified these after conducting more than 250 problem-solving interviews with students ranging in ages from kindergarten through 12th grade—in addition to interviews with teachers, mathematicians, and mathematics educators**. We found these WoR to be robust and consistent across grade levels!** And, perhaps most important, these WoR help us to structure our knowledge so that we can identify important differences in children's thinking about integer arithmetic. This helps us, as teachers, to better attend to and interpret student thinking in-the-moment.

These ways of reasoning are order-based, analogically based, computational, formal mathematical, and alternative.

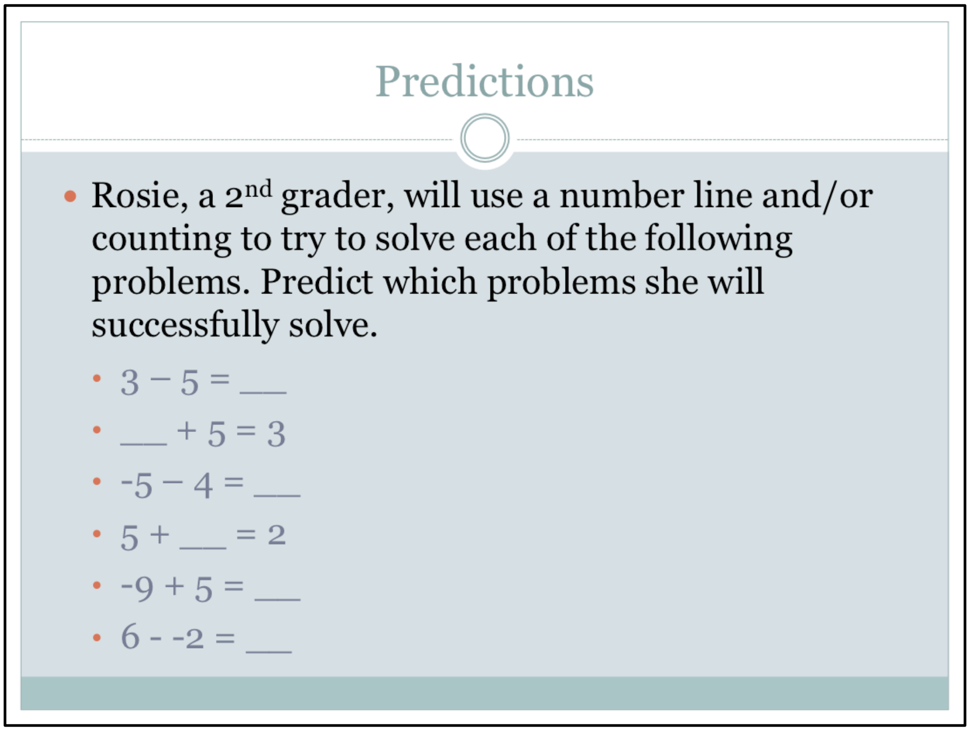
Our goal for the rest of our talk is to introduce you to these WoR, to engage you in thinking about children's reasoning collectively, and to spend time thinking about tasks that you might use in your classrooms, when you might use them, and why you might use them



Order-based Reasoning [2 minutes]

Order-based reasoning defined: Students leverage the sequential and well-ordered nature of numbers to reason about an integer task. Using an order-based way of reasoning, one places integers in a sequence and often uses a counting strategy or a number line with motion/movement.

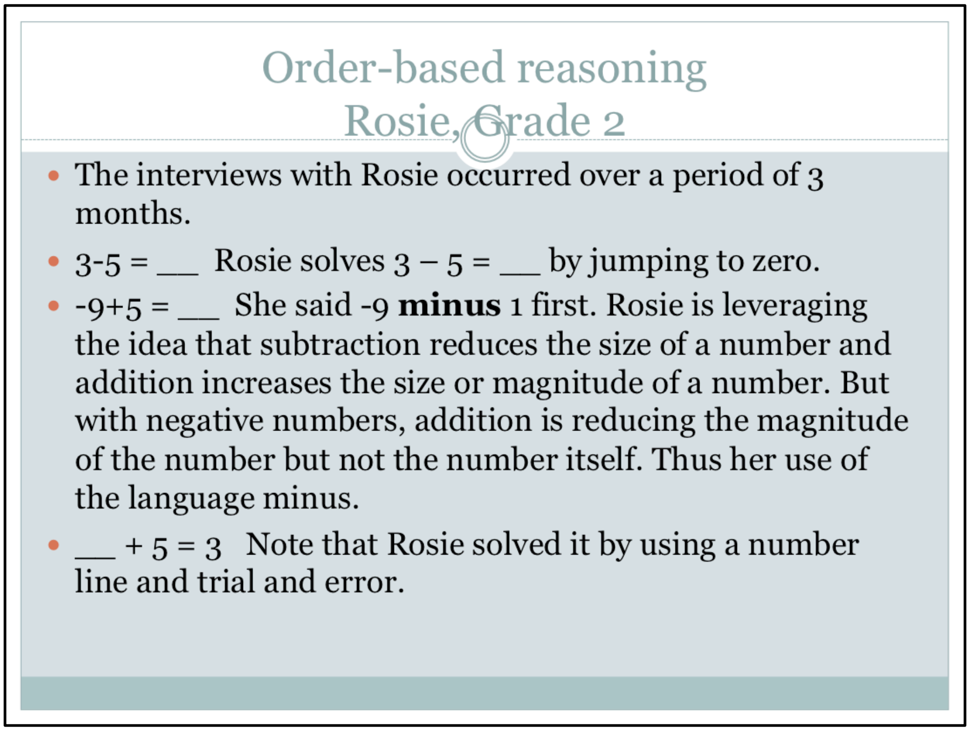
One of the strategies you may have identified was Liberty's counting strategy in counting up from -2 in her solution to -2 + __ = 4. This is an example of order-based reasoning. [Add in others generated from the beginning activity.]



Predictions for Rosie [3 minutes]

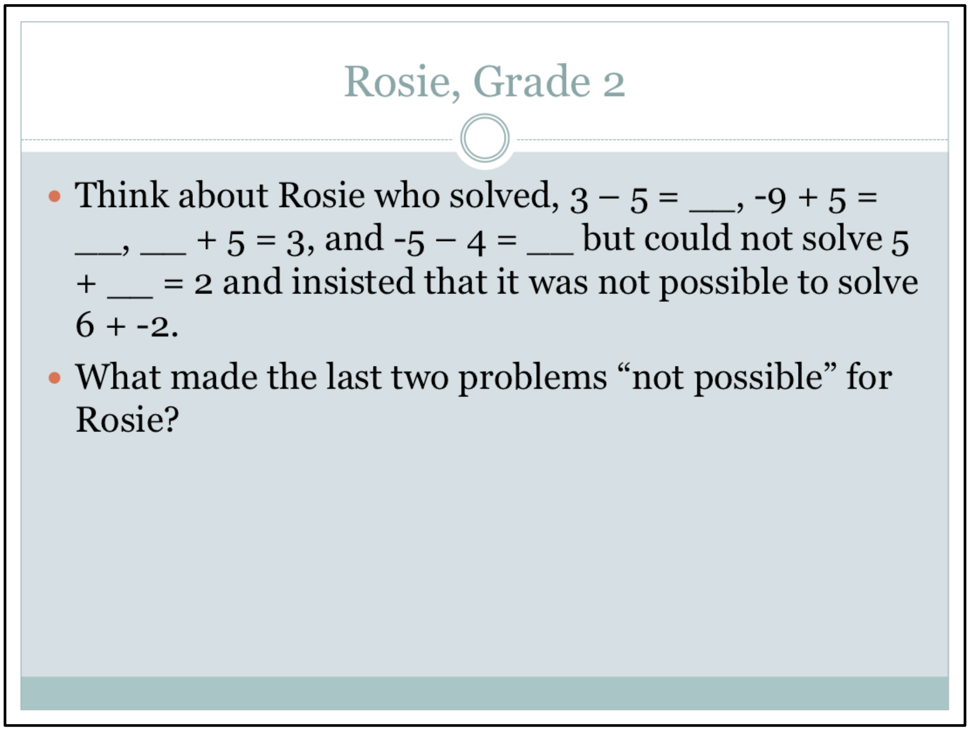
Now that you have a sense for what order-based reasoning may look like, we'd like you to think like a 2nd grader.
Spend a couple of minutes predicting how Rosie, a 2nd grader who consistently uses order-based reasoning might approach these problems. Negative numbers have not been taught in Rosie's 2nd-grade classroom.

Which problems can she solve and which ones might be particularly challenging? [Encourage them to draw a number line or to actually do some counting.]



Rosie Video [11 minutes]
Video clip itself is 5 minutes, 10 seconds long. Note that the interviews with Rosie occurred over a period of 3 months—the first interview occurred on a special occasion at school, that is why Rosie is wearing her pajamas.
3-5 = __
Rosie is first going to solve 3-5 = __. Play video and pause after she answers. Turn and talk with the person next to you: Reconstruct what you think Rosie did. Then, consider how this strategy involves order?
[1 minute] [Note that she solves 3 – 5 = __ by jumping to zero]
-9+5 = __
Now we'll watch Rosie work on the problem -9 + 5 = __ . Pause after her answer … Re-enact Rosie's counting strategy with your neighbor. [1 minute]
In your re-enactments, did anyone include the fact that she said -9 **minus** 1 first? (Show of hands.) Rosie's language alludes to an idea that we think is important when considering order-based strategies. Rosie is leveraging the idea that subtraction reduces the size of a number and addition increases the size or magnitude of a number. But with negative numbers, addition is reducing the magnitude of the number but not the number itself. Thus her use of the language minus.
__ + 5 = 3
The next problem Rosie will work on is __ + 5 = 3. Play clip. Pause and ask teachers to re-enact what Rosie did [1 minute]. [Note that she solved it by using a number line and trial and error]. Could make the connection that this form of 'trial and error' is related to what we see children doing with start-unknown problems when direct modeling

I'm going to show you the next three problems in a row. Please write these down: -5 – 4 = ____ , 5 + __ = 2, T/F 6 + -2 = 6 – 2. The first one will highlight Rosie's use of the number line, and you will see that Rosie has some difficulty with the last two, so pay attention to what 's going on. Play the next 3 clips in a row.



Interpreting Rosie's Thinking [5 minutes]
Why could Rosie solve some of these problems using order-based WoR, yet some problems "were not possible in her way of thinking." Spend 1 minute considering this question. And, yes, I realize that we are giving you 1 minute to think about what we've had 4 years to think about!

Share out conjectures from audience? [3 minutes]

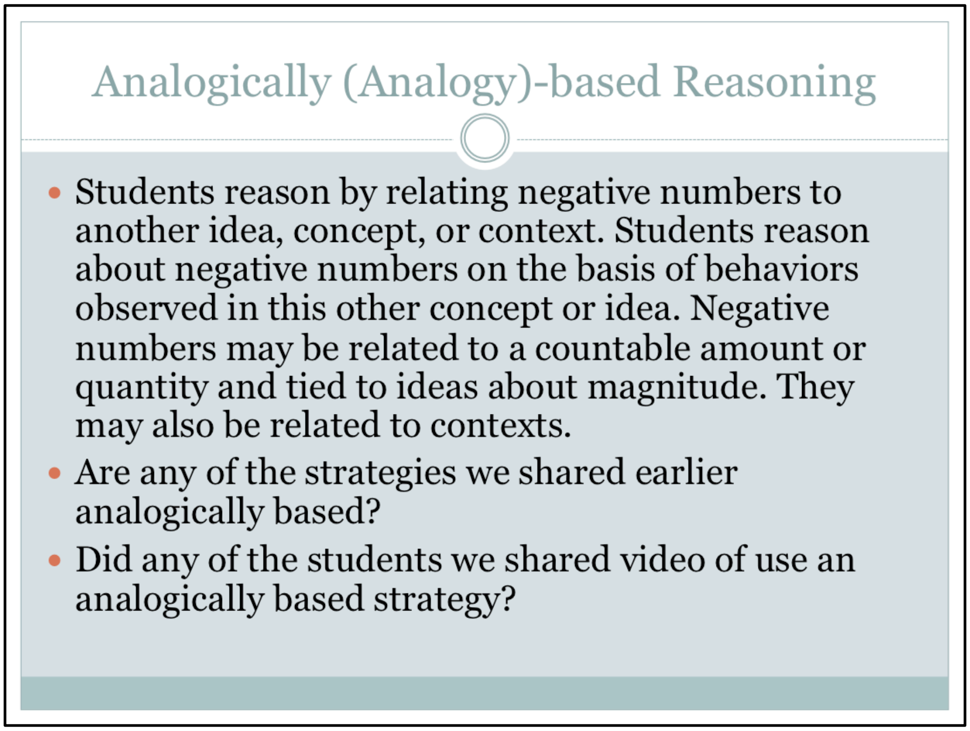
Wrap-up point. An issue one has to confront with an ordinal approach is assigning meaning to 'double signs'; in other words what does *adding* or *subtracting a negative number* mean? For a change-unknown problem, like 5 + __ = 2, students have to reason about how to start at 5, move right (b/c of addition) and yet end up to the left of the number at 2! That's hard!
What meaning or interpretation does Rosie have for adding or subtracting a negative number?
More generally, what are potential challenges to a number-line, counting, or order-based approach? Developing meaning for *adding* or *subtracting a negative number*.

Order-based reasoning

- Rosie, at the end of our problem-solving interviews

That's not the end of the story for Rosie [4 minutes]

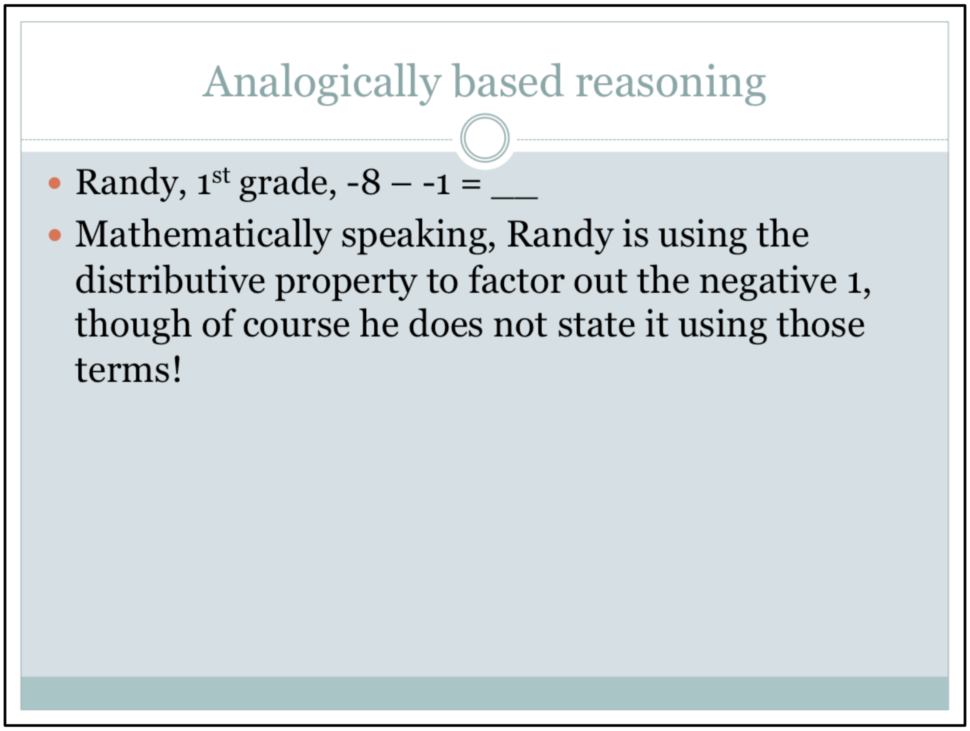
At the end of our interview, Rosie began to use the idea of negatives doing 'the opposite of' to reason about adding and subtracting negative numbers and change unknown problems. Rosie surprised us! We are going to share a clip with you where Rosie the problem 2 - __ = 6. Spend 30 seconds and solve these two problems on your own.
She will giggle because she realizes that something she earlier said was not possible, now feels, in some way, possible. Play the last Rosie clip [3 minutes]
When we pointed out to Rosie that she used to think these problems were impossible to solve, she shared with us that she had continued to think about these kinds of problems since our last interviews.



Analogically-based Reasoning [3 minutes]
Analogy-based (or analogical) reasoning defined: Students reason by relating negative numbers to another idea, concept or context. Students reason about negative numbers on the basis of behaviors observed in this other concept or idea. Negative numbers may be related to a countable amount or quantity and tied to ideas about magnitude. They may also be related to contexts. [Emphasize analogy]

Think about these questions:
Which of the strategies that we shared collectively used analogically based WoR?
Did any of the students we shared video of earlier use an analogically based strategy?

We consider Reynaldo's explanation of owing/debt and Noland's strategy of treating Negatives Like Positives to be examples of analogical reasoning.



Little Randy Video. [3.5 minutes]

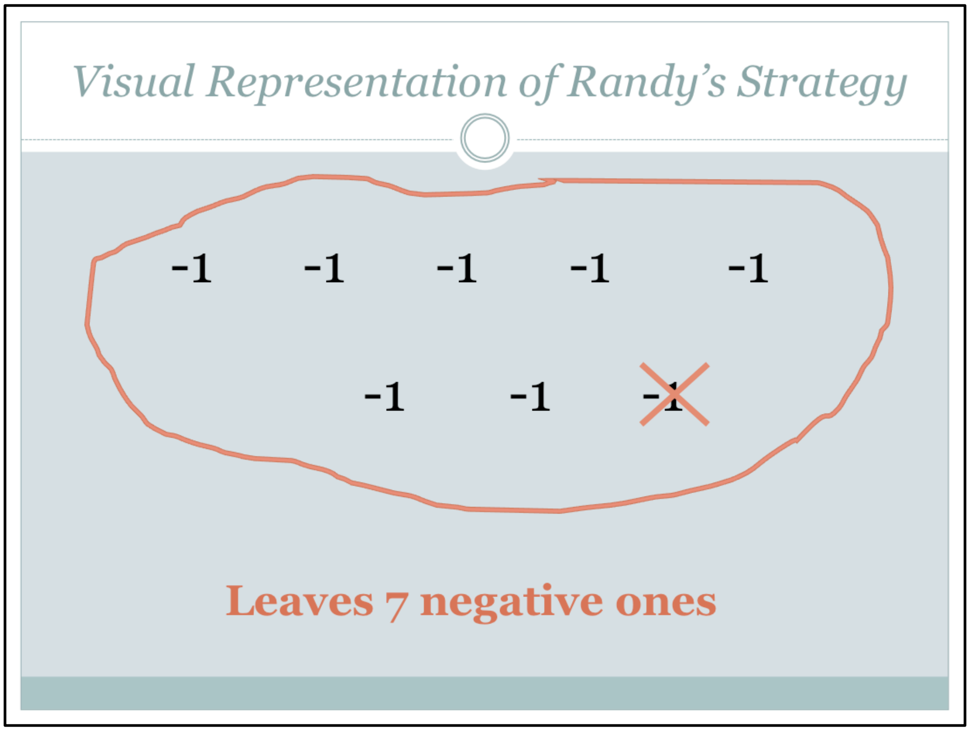
Here is an example of Analogical Reasoning.
Spend 30 seconds thinking about how you would personally solve -8 - -1 = __.
Did anyone do *boom boom*? If so, that would be a computational way of reasoning.

52 sec, Randy video clip of -8 – -1

Talk at your table and try to understand Randy's strategy [1.5 minutes]. Ask for audience interpretation of Randy's thinking.

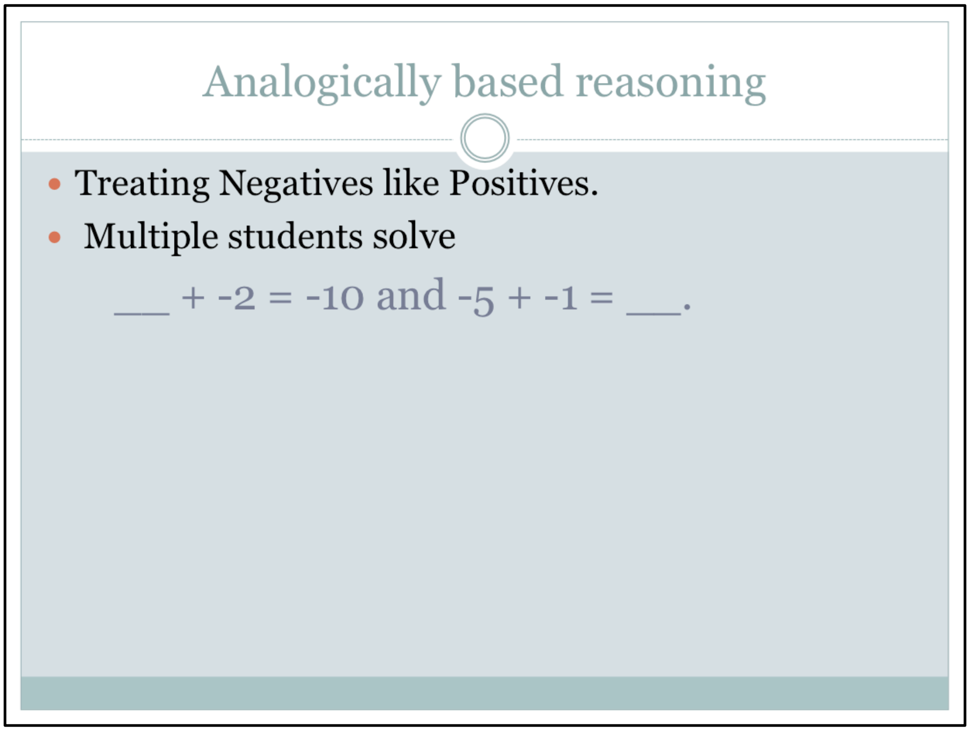
Mathematically speaking, Randy is using the distributive property to factor out the negative 1, though of course he does not state it using those terms! (see below, but I will not share with audience). Here is a visual representation of his thinking (go to next slide and play in order). In Linda Levi's talk yesterday, we saw powerful examples of students' using fundamental properties of algebra just as Randy does.

$-8 - -1 = 8(-1) - 1(-1) = (8-1)(-1) = 7(-1) = -7$. This is not unlike $8a - 1a = 8(a) - 1(a) = (8-1)(a) = 7a$.



Visual Representation of Randy's Strategy [30 seconds]
I have 8 negative 1s and I remove one negative 1. I now have seven negative 1s. Just like I can count oranges or unifix cubes I can count negative ones. You could imagine bear counters there instead of negative 1s.

So what?

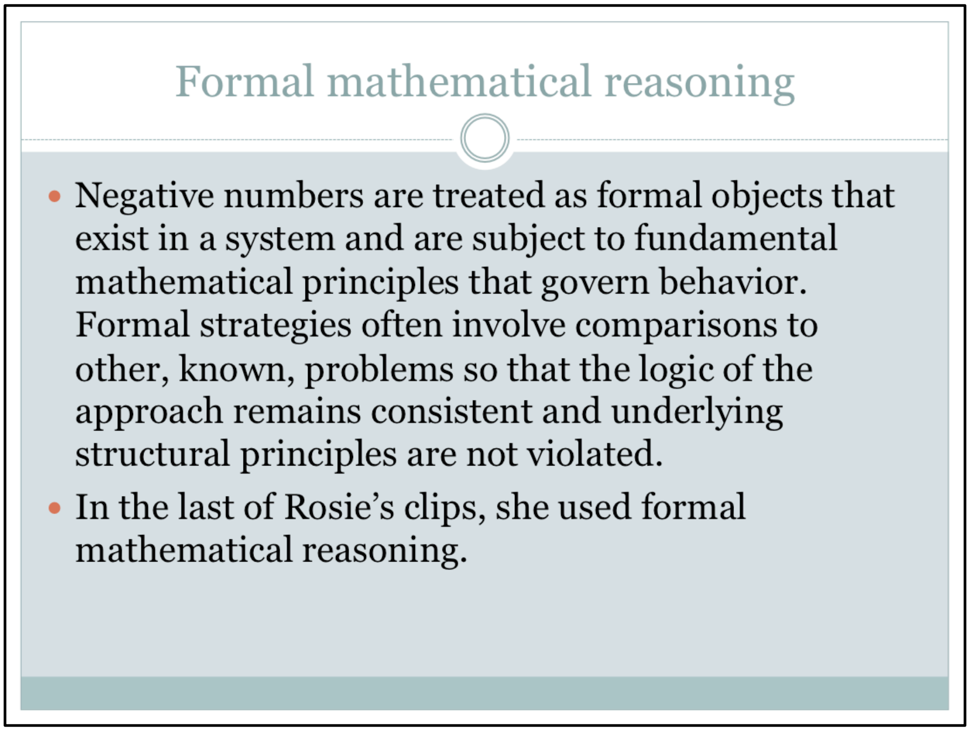


Negatives like Positives Montage (involves all grade levels, except maybe 2nd grade) [9 minutes]

You are going to watch students of all ages solving two problems using the same strategy, *Treating Negatives Like Positives*. The problems they are going to be solving are
__ + -2 = -10 and -5 + -1 = __. Take a minute and share how you are thinking about each problem with your neighbor. [1 minute 20 seconds]

Video is 2 minutes, 40 sec.

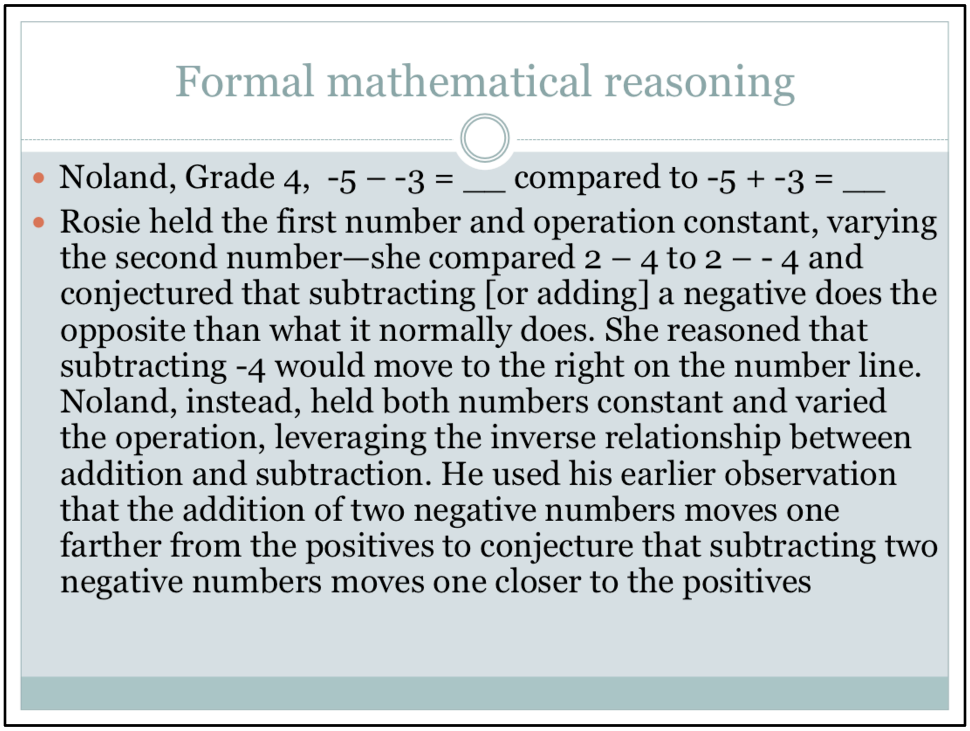
The basic argument is that I can reason about negatives by comparing them to how positives behave. Of course the underlying assumption is that they do behave similarly! Do they? At your tables, we'd like you to discuss this strategy. Do you understand how the students were solving these problems. Can you use this strategy to solve -6 – -2 = __. Can you use this strategy to solve 6 – -2? [3 minutes]
(Maybe) have one table share how they thought about -6 – -2 with the strategy of Treating Negatives Like Positives [1.5 minutes].



(4 minutes)
Let me turn this over to Randy for a historical connection.

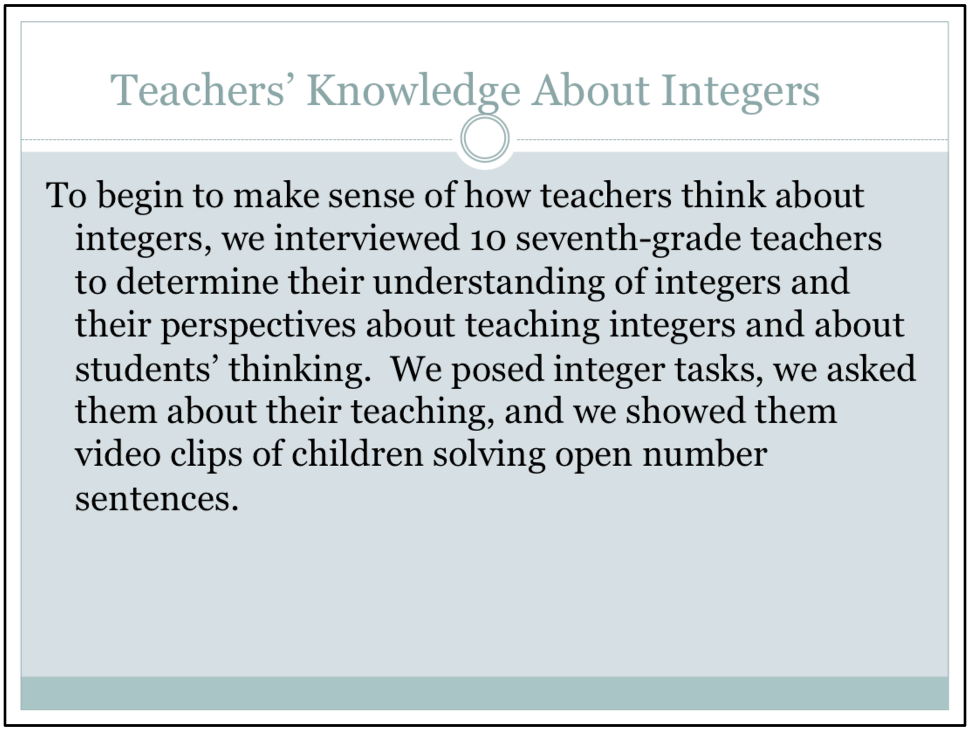
Randy: So earlier I talked about the difficulties mathematicians experienced historically with negative numbers. The history of mathematics is full of the need to extend our number systems from what we are comfortable with to something more complex. We started out counting sheep or children, and we needed only natural numbers. Geometry was based on magnitudes, which could not be negative. Fractions bothered us. Irrational numbers bother us. Even zero bothered us. Something happened in the 19th century that basically settled all this in, historically, a moment in time. That is, mathematicians decided to accept a formal approach to mathematics. Negative numbers had to be accepted because they were necessary to maintain and expand a consistent mathematical system, whether or not we liked how these numbers felt. And these formalisms, curiously enough, also can be found in students' reasoning. But just because mathematicians decided it does not mean children's ways of reasoning automatically shifted. They continue to struggle with these extensions the same way mathematicians did before the 19th century.

Formal Mathematical Reasoning, Jessica will read Linda's handout, Number and Quantity
Formal ways of reasoning defined: In this type of reasoning, students treat negative numbers like formal objects that are part of a mathematical system. Students may use fundamental mathematical principles, or a form of proof by contradiction. Strategies often involve comparisons to other, known, problems so that the logic of the approach remains consistent.



Example #2 of Formal Mathematical Reasoning [2.5 minutes]

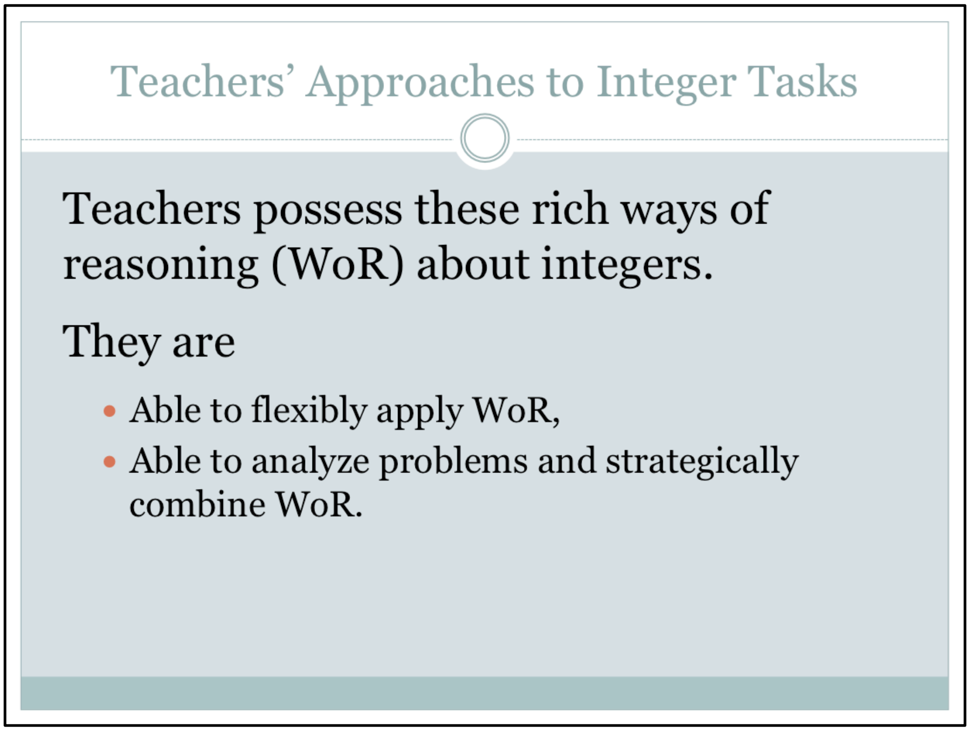
40 seconds of video
Rosie held the first number and operation constant, varying the second number—she compared 2 – 4 to 2 – - 4 and conjectured that subtracting [or adding] a negative does the opposite than what it normally does. So instead of moving to the left as done when subtracting 4, she reasoned that subtracting -4 would move to the right on the number line. The difference between Noland and Rosie was that Noland held both numbers constant and varied the operation, leveraging the inverse relationship between addition and subtraction. He used his earlier observation that the addition of two negative numbers moves one farther from the positives to conjecture that subtracting two negative numbers moves one closer to the positives



Our Integer study focused on the thinking of students, but to begin to make sense of how teachers are thinking about integers, we interviewed 10 seventh-grade teachers to determine their understanding of integers and their perspectives about teaching integers and about students' thinking. We posed integer tasks, we asked them about their teaching, and we showed them video of children.

What did we find?

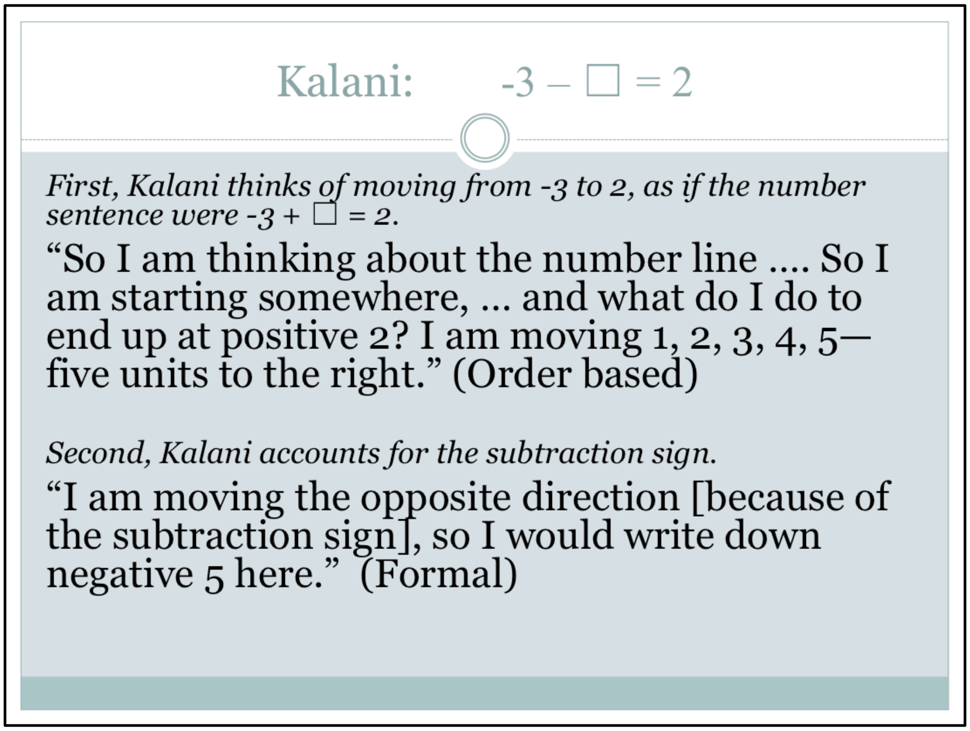
30 seconds



Teachers possess rich ways of reasoning, and they apply these ways of reasoning.

For example…

15 seconds



For example, consider Kalani’s approach to -3 – box = 2.
Highlight that first is ordered based, second is formal
Note that, even on tasks where the result was unknown, which is the classic type of integer operations task in middle school math, all but 1 used reasoning other than computational reasoning.

1 minutes

Interpreting Student Thinking

Difficulty understanding student reasoning

Kalani: "I think he got confused. There's no context involved .... So I don't see a clear understanding on his part at all." (referring to Noland's solving -5 + -1 = ☐)

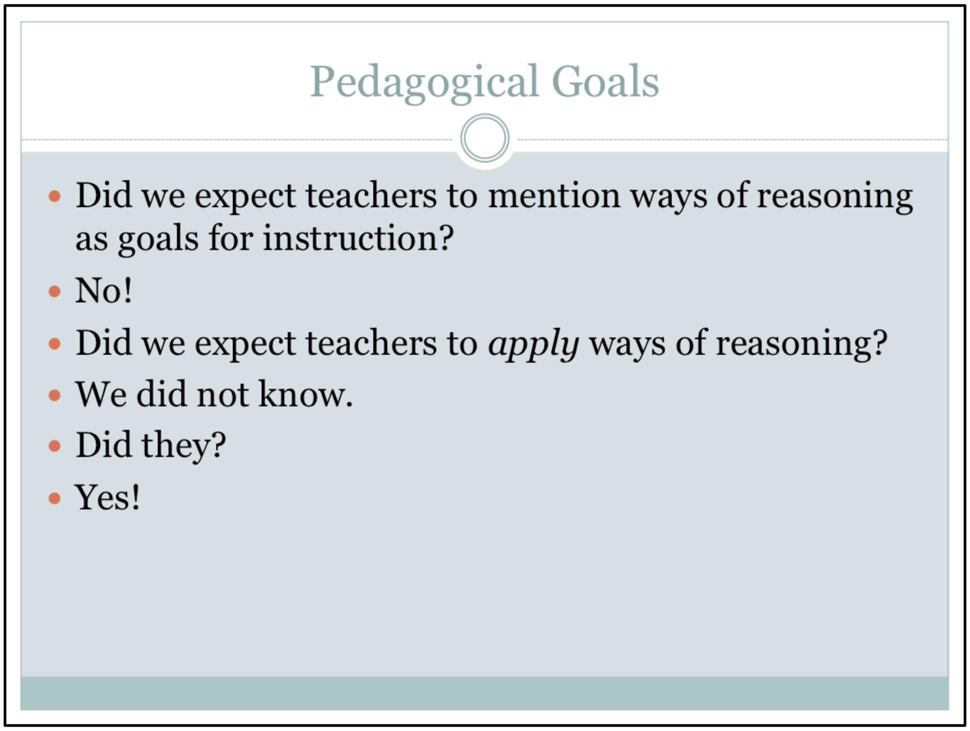
"I don't quite understand him when he used the opposite. Opposite of what? ...There is no context." (referring to Noland's solving -5 – -3 = ☐)

Interestingly, we showed Kalani a video of a child using what seemed to us to be identical reasoning to the reasoning Kalani used.

We think it is because he has not had the chance to reflect on and label these ways of reasoning.

If we had time, we would show you other examples of teachers who are unable to follow student's reasoning when they themselves used almost the same approach.

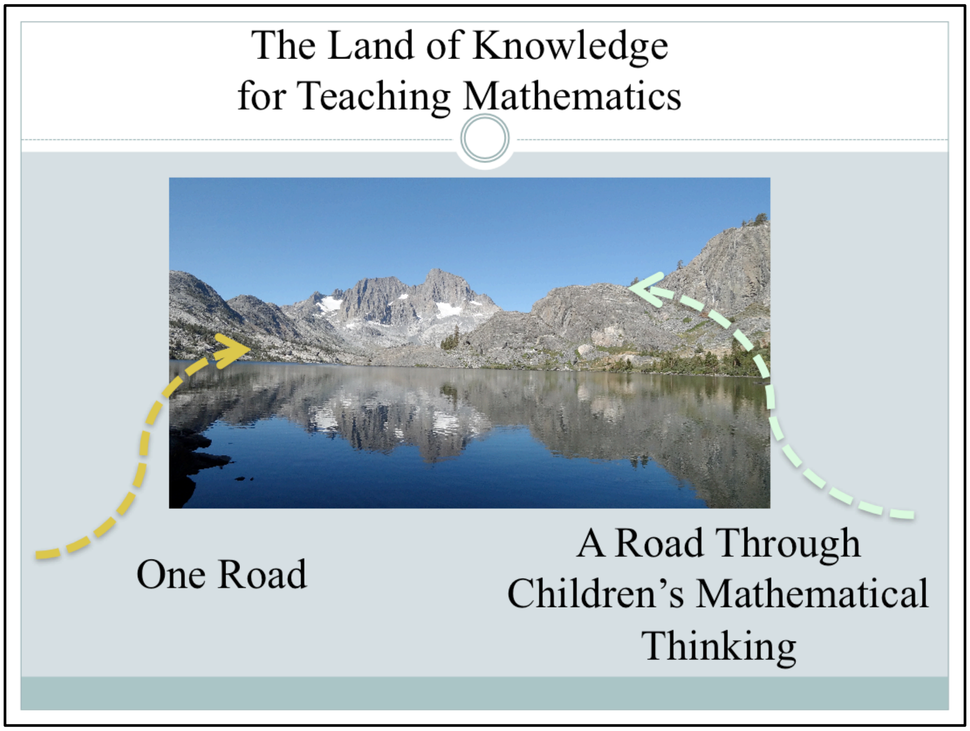
45 seconds



2:24–2:27 (Slides 24–28, pedagogical goals)
We are not here to be critical of anyone, of students, of teachers, of teacher educators, or of researchers.

Before this project began, we had not considered focusing integer instruction around these ways of reasoning.

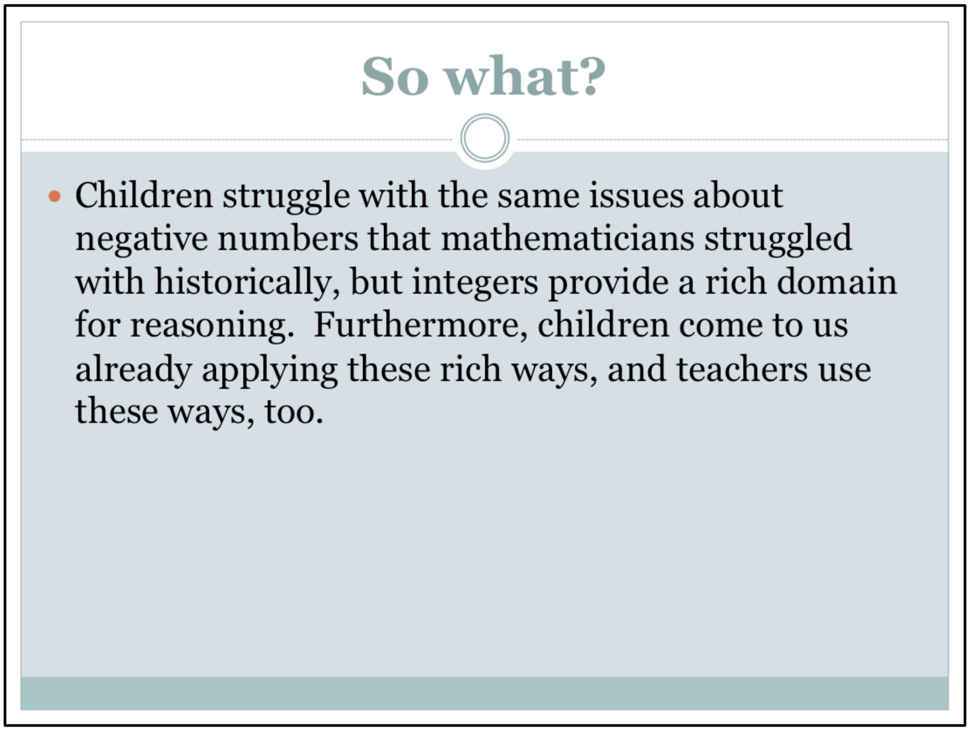
But for teachers to use knowledge in their teaching, we have found that not only must teachers hold the knowledge, but also the knowledge must be structured in some meaningful way.
1 minutes



Metaphorically, here is the land of knowledge of teaching mathematics.
Here is a road to that land. And from the bank, one has a view of that land.
But here is a second road, through children's mathematical thinking, and we suggest that this road leads to an overview that is broader, more expansive, and more complete. One sees more and hence is better poised to use that knowledge during instruction, when the road leads through the land of children's mathematical thinking.

And we believe that viewing the landscape of integers through the ways of reasoning that we shared today might help structure that knowledge for teachers, thereby making this knowledge accessible and useful when teachers are trying to make sense of their students' reasoning.

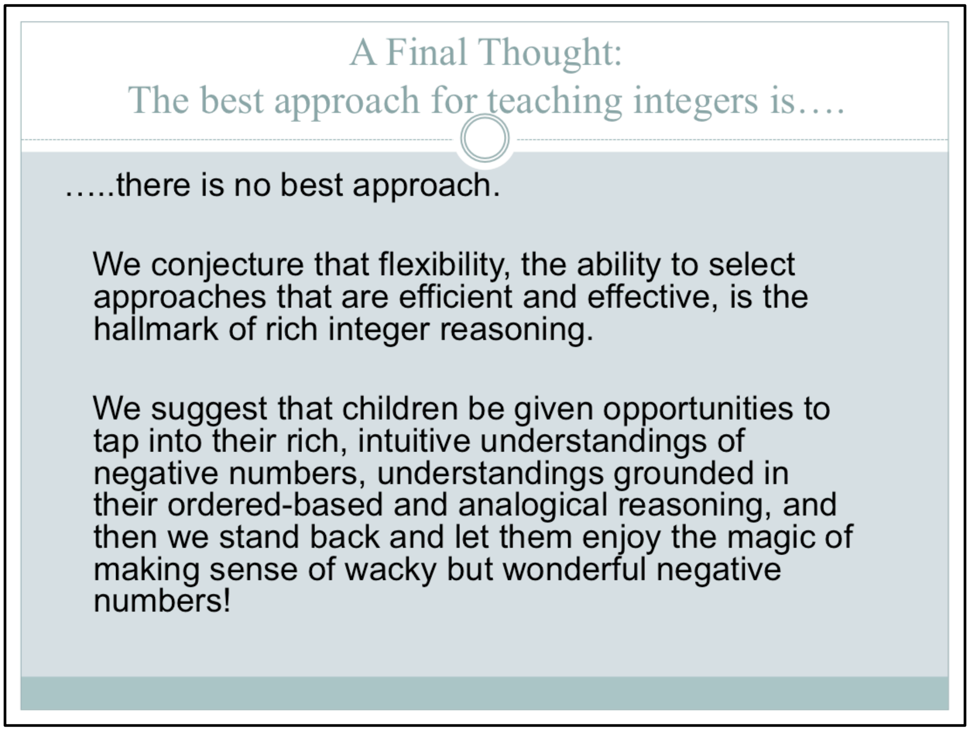
45 seconds



Again with this question? Ok, I guess we need to address this. Peg Smith talked Wednesday night about the goals we hold. We think that that is key. She gave an example of rich reasoning one might apply when solving a traditional-looking proportional reasoning task, and Linda Levi showed us examples of rich reasoning students use to solve traditional-looking algorithmic tasks. But what if I, a teacher, think that the purpose of proportional reasoning tasks, or whole-digit multiplication tasks, or integer operations, is solely to learn the procedures. As one student sung to us about integer addition,

"Same signs, add and keep
Different signs subtract
Keep the sign of the bigger number
Then you'll be exact."

Children struggle with the same issues about negative numbers that mathematicians struggled with historically, but integers provide a rich domain for reasoning, and not only that, but children come to us already applying these rich ways, and teachers use these ways too.



No holy grail for teaching integers ....

Do you want to use chips? Go ahead. Do you want to use money? Go ahead. Do you want to sing songs? Go ahead. Do you want to use boom boom? Go ahead. I just thought of something. How about if tonight at 9 in the bar we hold a meeting of the Boom Boom Support Group? Hm, how am I going to explain that to my wife?

But there is a bigger question to ask. What ways of reasoning can we support students in developing while they engage, not just with integers, but with proportional reasoning, and whole number operations, and geometry, and algebra, and ....

And finally, Jessica and I want to acknowledge how wonderful it is to be here, with you. If I, as a school teacher many years ago, had come to this conference, I might have felt, simultaneously, inspired and overwhelmed—inspired by all the wonderful ideas, and overwhelmed because I know that I can't do all this immediately. So Jessica and I want to tell you to be patient and recognize that meaningful change takes time; it is not linear, but it is a glorious endeavor, and we appreciate the professionalism that leads you to want to continue to think about how to become even better at your craft.

So thank you.

Discussion