Project Team

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Project Z
Mapping Developmental Trajectories of Students' Conceptions of Integers

3-Year Project (2009–2012)

Year -1 | Year 0 | Year 1

"There are other numbers under zero. I know about them, but I don't know what they're called." (Lucy, 1st grade)
"Negative numbers aren't really numbers. They're just acting like other numbers." (Violet, 2nd grade)

Research Questions

1. What are students' conceptions of integers and operations on integers?
2. What are possible developmental trajectories of students' understandings?

Why Study Integers?

- Integers mark a transition from arithmetic to algebra because of their abstract nature (negative numbers have no concrete embodiments) and because students must understand fundamental algebraic principles, for example, using additive inverses, which first come into play with the introduction of integers.
- Difficulties in algebra have been linked to a lack of integer understanding (see, e.g., Moses, 1989).
- Students have great difficulty operating on integers, and those difficulties appear to be robust (see, e.g., Thomaidis & Tzanakis, 2007; Vlassis, 2002); even those who have completed algebra courses are challenged by problems with negative numbers (Reck & Mora, 2004; Vlassis, 2002). In particular, students have difficulty flexibly moving among the three meanings of the – sign, as a binary operator, a sign as part of the number, and the negation of a quantity or expression.
- When compared with the literature on rational numbers or place value, literature on students' understanding of integers is relatively sparse.

Interviews With Four Groups

Grade Level	Number of Participants	Rationale
2 and 4	40 per grade	To track informal conceptions
7	40	To track typical conceptions after the majority of formal instruction
Students Enrolled in Precalculus or Calculus	40	To track conceptions of successful college-bound students long after formal instruction
Specialized Adults	30	To track expert conceptions from one of four perspectives ➢ historical mathematical ➢ formal mathematical ➢ mathematics education ➢ teacher

Work to Date

- Developing and piloting assessment items
- Conducting more than 65 pilot interviews across K–12 and 4 specialized-adult interviews
- Preparing for major data collection for Spring 2011; developing preliminary coding schemes and interview protocols; finalizing interview items
- Disseminating preliminary findings at conferences and in publications/proceedings (see handout)

Open Number Sentences Versus Contextualized Problems

- We have chosen to emphasize *open number sentences* because they (a) are consistent with the abstract nature of negative numbers and (b) are sites in which students can solve problems with familiar whole numbers in the problem statement and negative numbers as solutions (for example, 5 + ☐ = 3 or 6 – ☐ = 8).
- We have chosen to deemphasize *contextualized problems* because we see few contexts with solutions that *necessitate* the explicit use of negative-integer reasoning to obtain solutions.
- We suggest that integers might be best understood within the structure of mathematics and not within the realm of contexts.

Two Primary-Grade Examples: What Are Violet's and Sammy's Conceptions of Negative Numbers?

Violet, 2nd grade

☐ + 5 = 3

-5 – 4 = ☐

True/False
6 + -2 = 6 – 2

Sammy, 1st grade

-7 – ☐ = -5

-5 + -2 = ☐

1 + -2 = ☐

☐ + 5 = 3
Violet used trial and error on the number line, starting at -5 and counting up five places to end at 0, then starting at -4 and ending at 1. After starting at -3 and ending at 2, she answered "Negative two" and counted up five places to end at 3.

-7 – ☐ = -5
"Negative two. You just do … 7 minus 2 equals, and the 5, because that's the answer for real numbers. Okay, so I just added a negative to all of them, and there is my answer."

-5 – 4 = ☐
"Negative 9. I started at negative 5 and I went back 4. (On the number line, Violet counts by ones from -5 to -9.) One, 2, 3, 4." She explained that she knew to go to the left because "it is subtracting."

-5 + -2 = ☐
(Sammy writes -7). "It's like 5 plus 2 is 7 but you're doing negatives."

Tr ue or False? 6 + -2 = 6 – 2
"False. Because that's a negative (pointing to -2 in 6 + -2), and you can't add with negative numbers." (Violet later confirmed that adding negative numbers was impossible in her way of thinking.)

1 + -2 = ☐
Sammy explains the problem cannot be done "because this one (points to 1) would have to be a negative number. If you did one plus a negative number that wouldn't even be a number … They're like magnets that push apart from each other."

Outcomes/Deliverables

- A framework to identify problem types and increasingly sophisticated problem-solving strategies, as related to student thinking about integer operations.
- A valid and reliable paper-pencil integers assessment and set of scoring rubrics that measure a variety of students' understanding of integers.