Project Team

Lisa Lamb Lisa.Lamb@sdsu.edu

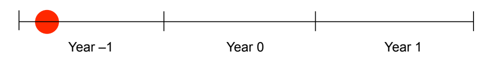
Randolph Philipp rphilipp@mail.sdsu.edu

Jessica Pierson jpierson@mail.sdsu.edu

Bonnie Schappelle, Candace Cabral, & Ian Whitacre

Project $\mathbb{Z}$
Mapping Developmental Trajectories of Students' Conceptions of Integers

3-Year Project (2009–2012)



When, as a young boy, mathematician Jerry King (1992) told his teacher that he did not understand why −2 multiplied by −3 gives +6, she answered sternly, "You have a very bad attitude toward mathematics. I've already told you that the product of two negative numbers is always positive" (p. 277). *

Research Questions

1. What are students' conceptions of integers and operations on integers?
2. What are possible developmental trajectories of students' understandings?

Why Study Integers?

- Integers mark a transition from arithmetic to algebra because of their abstract nature and because students must perform algebraic procedures using additive inverses, which first come into play with the introduction of integers.
- Difficulties in algebra have been linked to a lack of integer understanding (see e.g. Moses, 1989).
- Students have great difficulty operating on integers and those difficulties appear to be robust (see, e.g., Gallardo, 1995, 2002; Vlassis, 2002); even those who have completed algebra courses are challenged by problems with negative numbers (Reck & Mora, 2004; Vlassis, 2002).
- When compared with the literature on rational numbers or place value, literature on students' understanding of integers is relatively sparse.

Interviews With Four Groups

Grade Level	Number of Participants	Rationale
K–4	40	To track informal conceptions
5–7	60	To track typical conceptions in years integers are usually taught
8–12	20	To track conceptions after formal teaching of integers
Specialized Adults	30	To track expert conceptions from one of 4 perspectives ➢ historical mathematical ➢ formal mathematical ➢ children's thinking ➢ teacher

Outcomes/Deliverables

- A framework to identify problem types and increasingly sophisticated problem-solving strategies, as related to student thinking about integer operations.
- A valid and reliable paper-pencil integers assessment and set of scoring rubrics that measure a variety of students' understandings of integers.

Why Are Integers Difficult to Understand?

One might consider

a) the lack of a concrete entity from which to abstract the idea of negative number (consider the difficulty historically and the reluctance of the Western mathematics community to accept negative numbers as entities);

b) the ways that integers are symbolically represented (that is, that the negative sign [–] has two meanings: one as an operation (subtraction) and the other as a negative number); and

c) the unintuitive models currently available for supporting operations on integers.

What Might These Responses Tell Us About a Child's Understanding of Negative Numbers?

1)

A child wrote the following to solve the problem 47 – 29: **47** **– 29** **20** **– 2** ⬅ **18**	A child wrote the following to solve the problem 47 – 29: **47** **– 29** **20** ➡ **– 2** **18**
How might this child have reasoned about the –2? I can't take 9 from 7, but I can take 7 of the 9 from 7, leaving 2 more to take from the 20, so 20 minus 2 is 18.	How might this child have reasoned about the –2? 20 plus –2 is 18.

2) Children's responses to –5 + 8

a) Counting on. "Using the number line, I counted-on 8 spaces starting at –5 and landed on 3."

b) Decomposition and 0 as a benchmark. (Breaking down 8 to 5 + 3), "I know that –5 + 5 is 0 and 3 are leftover, so the answer is 3."

How Can You Help Us?

- Share your integers tasks for use across grades K–12 and adults.
- Join our conversation about integers.

* See handout for references