Making Sense of the Counterintuitiveness of Integers Without Simply Relying on Rules

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In this session, my goal is to make more explicit the difficulties students (and perhaps we as teachers) have with integers as well as highlight a few key items to think about in your instruction of integers to build on students' reasoning and help them avoid possible pitfalls.

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To reflect on our practice and how we think about integers, let's consider a few questions about experience with integer instruction.

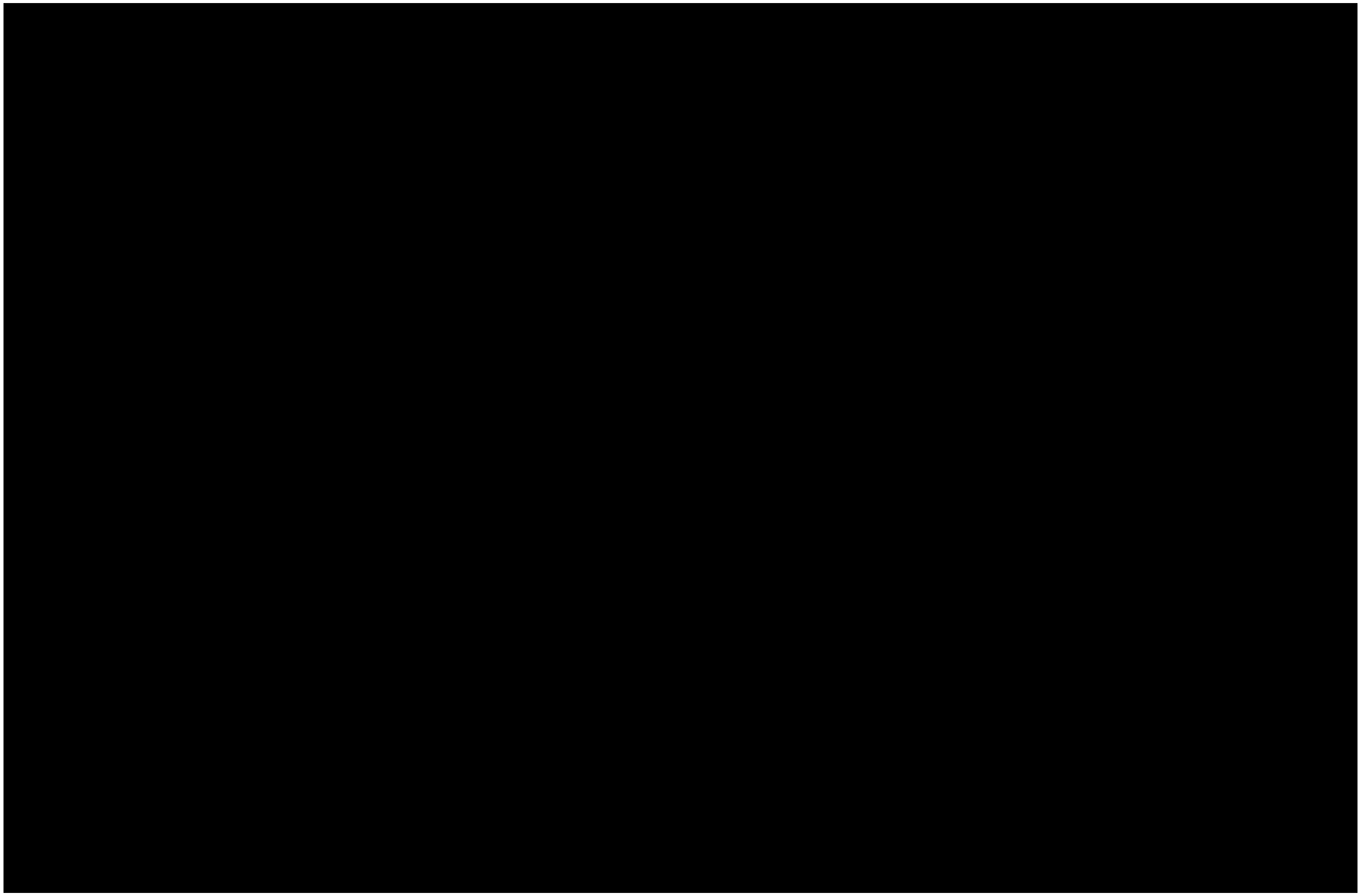
1. What grade do you think is best for introducing integers?

2. Do you have a particular way to teach integers? Do you use more than one way? Have you found limitations to your model? Do you believe that there is a best way?

3. What difficulties with integers have you observed in your students?

Students and Rules

6 – -2



Historically, mathematicians struggled to make sense of numbers less than zero.

- Have you ever wondered how long negative numbers have been around?
- Diophantus (3rd century, Greece) claimed that the equation $4x + 20 = 4$ was "absurd" because the result 4 was less than the 20 units that were added.
- John Wallis (English, 17th century): "It is not possible that any magnitude can be less than nothing or any number fewer than none."
- Blaise Pascal (French, 17th century): "I know some who cannot understand that to take four from nothing leaves nothing."

What makes Integers so difficult?

5 + _____ = 1

$$\begin{array}{r} 63 \\ \underline{-\ 25} \end{array}$$

"You can't subtract 5 from 3, so …"

-5 + -4 = -9

Comparing integers

-5 __<___ 3

|-5| > |3|

Context and Negative Numbers

Yesterday you borrowed $8 from your friend to buy a school t-shirt. Today you borrowed another $5 from the same friend to buy lunch. What's the situation now?

Elementary Mathematics Is Not So Elementary!

a) Why is $4 - (-3) = 4 + 3$

b) Which is bigger, $-x$ or x?

c) What is the role of the minus signs in the following problem

$$-2 - x = 5$$

$$-x = 7$$

$$x = -7$$

Three interpretations of the – sign.

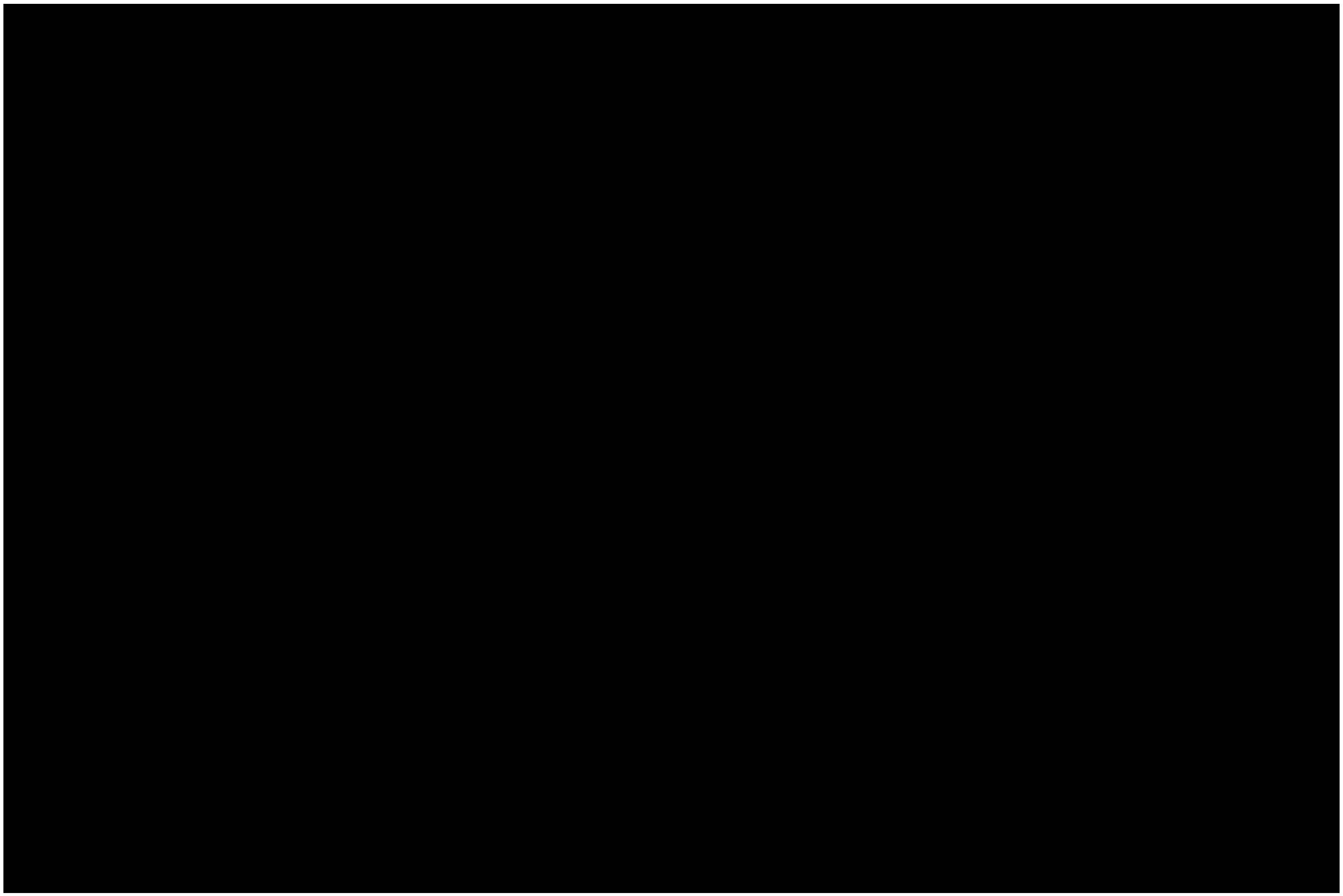
10 – 6	**“10 minus 6”**	Binary operator: **Student views the minus sign as involving two inputs and one output.**
– 5	“the opposite of 5”; “the negative of 5”	Unary operator: Student views the minus sign as taking *the opposite of* or *the negative of* 5.
– 5	“Negative 5”	Non operator: Student views the minus sign as *part of the number, not as the opposite of something.*

Wait!

If making sense of integers is so difficult, how can we help our students reason about them?

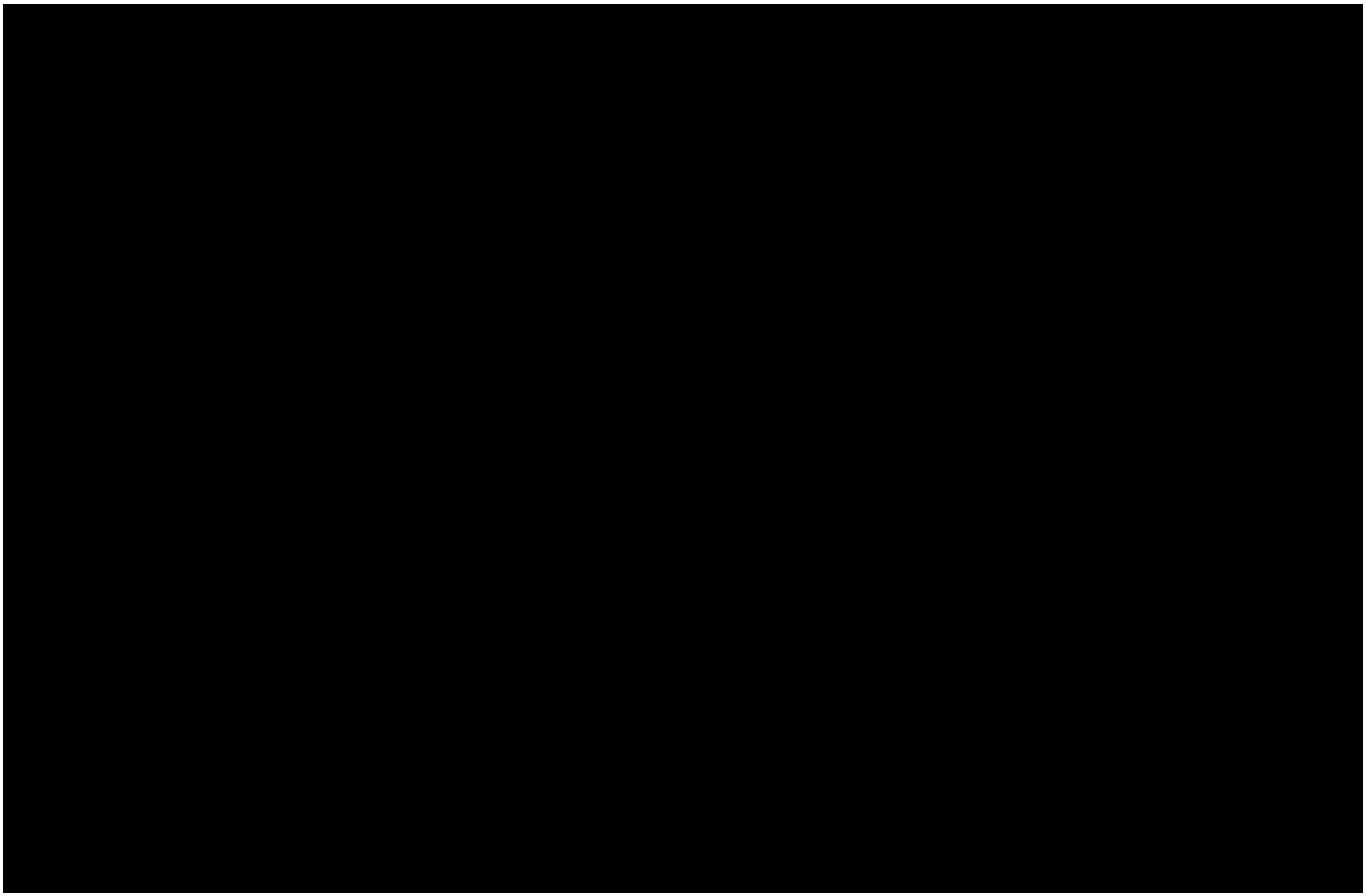
Children Making Sense of Integers

__ + 5 = -3, -5 – 4 = ___ , 3 – ___ = -2



An Example of Rich and Flexible Reasoning Andrew, Grade 11

3 – 5, 5 –_ = 8, -5 +_= -2, _ – 5 = -1, -8 – _= -2



Research Findings With Teachers

The overwhelming majority of teachers we interviewed said that they focus simply on procedural fluency. They themselves demonstrated a wide variety of techniques and kinds of reasoning of which they were unaware.

Big Ideas

- Negative numbers are not natural (either to us or historically to mathematicians). At the same time, students possess ideas that can be leveraged to help them reason about integers.

- Teachers tend to approach integers with things we want students to do, not with ways we want them to reason.

- There is no best way to teach integers. Different problems invite different strategies. We need to develop flexible thinking.

- Keys items to make explicit in instruction

  a) Number line b) Magnitude vs. ordering

  c) Additive inverse d) Multiple meanings of the minus sign

The CCSS Mathematical Standards

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision (in language and mathematics).
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.

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Transferring Ideas to Practice

List a few things that you might do differently with students to develop their reasoning with integers.

Speaker Evaluation

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	2	3	4	5

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" 31369 ___ ___ ___ __________ "

- Speaker was well-prepared and knowledgeable
- Speaker was engaging and an effective presenter
- Session matched title and description in program book
- Other comments, suggestions, or feedback

Example: " 31369 545 Great session! "