An Analysis of Mathematical Content Knowledge for Teaching*

John (Zig) Michael Siegfried
James Madison University
Lisa Clement Lamb
San Diego State University
Randolph Philipp
San Diego State University
Jessica Bishop
University of Georgia
Victoria Jacobs
University of North Carolina at Greensboro
Robert Joseph Nanna
University of Massachusetts Dartmouth
Casey Hawthorne
San Diego State University

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Participant group		Description
Prospective Teachers (PSTs) (n = 35)		Undergraduates enrolled in a first mathematics content course for elementary school teachers
Experienced Practicing Teachers		
	Initial Participants (IPs) (n = 31)	Experienced K–3 teachers who were about to begin sustained professional development focused on children's mathematical thinking
	Advancing Participants (APs) (n = 31)	Experienced K–3 teachers who had engaged with sustained professional development focused on children's mathematical thinking for 2 years
	Emerging Teacher Leaders (ETLs) (n = 32)	Experienced K–3 teachers who had engaged with sustained professional development focused on children's mathematical thinking for at least 4 years and were beginning to engage in at least minimal activities to support other teachers
Strong Mathematics Students (SMSs) (n = 32)		STEM students, with no teaching intentions, enrolled in upper–division mathematics courses

Note. All practicing teachers had at least 4 years of teaching experience (with a range of 4–33 years), and the number of years of teaching experience in each group averaged 14–16 years.

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The opinions expressed in this presentation do not necessarily reflect the position, policy, or endorsement of the supporting agency.

Group	Andrew	Ones
PST		
IP		
ETL		
SMS		

Your prediction

Group	Andrew	Ones
PST		
IP		
ETL		
SMS		

STEP Result

STEP Content Tasks

Ones Task

Below is the work of Terry, a second grader, who solved this addition problem and this subtraction problem in May.

Problem A

$$\begin{array}{r} {\scriptstyle 1} \\ 259 \\ +\ 38 \\ \hline 297 \end{array}$$

Problem B

$$\begin{array}{r} 3 \\ {\scriptstyle 1}\not{4}29 \\ -\ 34 \\ \hline 395 \end{array}$$

- Does the 1 in each of these problems represent the same amount? Please explain your answer.
- Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in problem B) 10 is added to the 2.

Andrew Task

In March, Andrew, a second grader, solved 63 - 25 = ☐ as shown below.

$$\begin{array}{r} 63 \\ -25 \\ \hline -2 \\ 40 \\ \hline 38 \end{array}$$

- Explain why Andrew's strategy makes mathematical sense.
- Please solve 432 - 162 = ☐ by applying Andrew's reasoning.

Strategies Task

a) Please provide solution strategies—as many as you can—that you might expect children to use to solve the following problem:

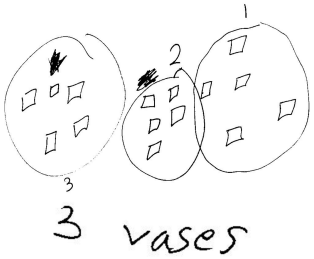
Pablo read 15 pages of his library book on Saturday. The book has 32 pages. How many pages will he have to read on Sunday to finish his book?

b) Circle the strategy or strategies that you would most likely see first graders using.

Division Task

The teacher needs to put 15 flowers in vases. Each vase can hold 5 flowers. How many vases does she need?

Russ, a first grader, solved this problem in February. To solve the problem, he counted out 15 linking cubes. He pulled out a group of 5 cubes and then another group of 5 cubes and then another group of 5 cubes, so all the cubes were gone. Then he counted the number of groups, "1, 2, 3," and said that 3 vases were needed. (Below is his record of his work.)



Please think about the following problems and whether Russ is likely to use similar reasoning when solving them. (Due to space constraints, below we present only the first of three parts. Parts 2 and 3 are similar but use different problem types.)

Part 1 There are 20 children on the playground. The teacher wants to play a game with 4 teams. How many children will be on each team if each team has the same number of children?

- To solve this problem, is Russ likely to use reasoning similar to the reasoning he used to solve the vases problem? ☐ yes ☐ no Why or why not?
- If you answered *no*, describe one strategy a young child would be likely to use to solve this problem.

Pat Task

In May, a teacher provided the following situation in her third-grade class:
I was at a store, and I saw that chocolate kisses come in bags of 42. I wanted to share these kisses among 7 people. How many kisses would each person get?

Following are the steps Pat told his teacher he had performed mentally to solve the problem. The teacher's follow-up questions confirmed that Pat's steps reflected a deep understanding of the problem situation.

$4 \times 10 = 40$

That is three 4s too many, so I have 12 left over.

$12 + 2 = 14$

$14 \div 2 = 7$

$4 + 2 = 6$. So $42 \div 7 = 6$.

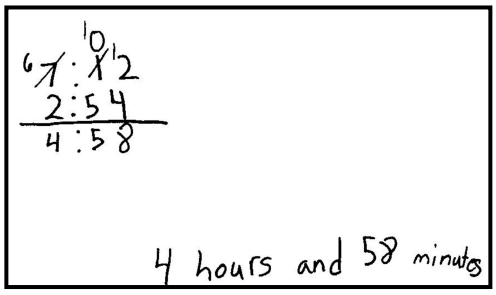
a) Please explain how each of Pat's steps makes mathematical sense in this context.
b) Use Pat's approach to solve $56 \div 8$.

Time Task

(This web-based task was administered so that respondents submit the response to Part 1 before responding to Part 2, and they submit the response to Part 2 before responding to Part 3).

Last month the children in a third-grade classroom were given the following problem: ***You were on a train that left Los Angeles at 2:54 p.m. and arrived in Phoenix at 7:12 p.m. How long were you on the train?***

Matt solved the problem as follows:



Matt explained,

"I couldn't subtract 4 from 2, so I borrowed 10 to make 12; 4 subtracted from 12 is 8. Then I couldn't take the 5 from the 0, so I borrowed 1 from the 7 and put it by the 0. Then I subtracted 5 from 10; that's 5. Then I subtracted 2 from 6, and that's 4. The answer is 4 hours and 58 minutes."

Time Task (Part 1 of 3)

1. What do you think about Matt's solution?

Time Task (Part 2 of 3)

Laverna, one of Matt's classmates, used a different method to solve the same problem that Matt had solved by subtracting. She "counted up" the elapsed time from 2:54 p.m. to 7:12 p.m.

2. If Laverna answered correctly, what was her answer?

Time Task (Part 3 of 3)

3. The class discussed Matt's subtraction solution and LaVerna's counting-up solution. They saw that Matt's answer was 40 minutes too large. Can you explain why Matt's procedure does not make mathematical sense?

4. If Matt uses his same procedure on another travel-time problem that also requires regrouping from the hours, will his answer again be 40 minutes too large?

5. Why or why not?

Jorge revised Matt's subtraction procedure and used it to calculate the elapsed time from 4:55 p.m. to 8:19 p.m. His answer was 3 hours and 24 minutes, the correct answer.

$$\begin{array}{r} {\scriptstyle 7\ 6} \\ 8\!\!\not{0}:19 \\ -\ 4:55 \\ \hline 3:24 \end{array}$$

6. What does the 7 represent in Jorge's revised procedure?
7. What does the 6 represent in Jorge's revised procedure?
8. How many minutes are represented within the oval in Jorge's procedure, shown below?

$$\begin{array}{r} {\scriptstyle 7\ 6} \\ 8\!\!\not{0}:19 \\ -\ 4:55 \\ \hline 3:24 \end{array}$$