Using Integers to Rethink the Role of Context in School Mathematics

Jessica Bishop, University of Georgia
Randy Philipp, San Diego State University
Ian Whitacre, Florida State University

Discussant: Michelle Stephan, University of North Carolina, Charlotte
Discussant: Vicki Jacobs, University of North Carolina, Greensboro

AMTE 2013

This presentation is based upon work supported by the National Science Foundation under grant number DRL-0918780. Any opinions, findings, conclusions, and recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSF.

Consider the following integer contexts:

Yesterday you borrowed $8 from a friend to buy a school t-shirt. Today you borrowed another $5 from the same friend to buy lunch. What is the situation now?

Imagine that you are standing on the beach. There is a bird flying 20 feet above the surface of the water and a fish swimming 5 feet below the surface of the water. How many feet higher is the bird than the fish? (If necessary, revise the question and pose, "How many feet apart are the bird and fish?")

- Solve each, write a number sentence that represents the context, and discuss how a 7th (or 11th or 4th) grader might solve each.

Outline of Presentation

- Solve/discuss 2 context problems
- Project Z background
- Why contexts?
- Findings and related video clips
- Michelle's thoughts
- Vicki's thoughts
- Audience discussion

Project Z: Mapping Developmental Trajectories of Students' Conceptions of Integers

Lisa Lamb, Jessica Bishop, Randy Philipp

Ian Whitacre, Bonnie Schappelle, Mindy Lewis

Candace Cabral, Jenn Cumiskey, Kelly Humphrey

This material is based upon work supported by the National Science Foundation under grant number DRL-0918780. Any opinions, findings, conclusions, and recommendations are those of the authors and do not necessarily reflect the views of NSF.

Project Z Study Design

- What are K–12 students' conceptions of integers, and what are possible learning trajectories?
- Our work is grounded in a children's thinking perspective; that is, we are interested in the rich and varied approaches that students use to make sense of integers and integer arithmetic.
- We conducted 160 problem-solving interviews using a cross-sectional design:
  - 40 students from each of Grades 2, 4, 7, & 11 across 11 ethnically diverse school sites.
  - Majority of questions were open number sentences (e.g., 3 – 5=__ or -3+__ = 6).
  - Today we report findings from context. problems.

Why Use Contexts?

- Many researchers, research traditions, and curricula within mathematics education speak to the importance of contexts in learning, particularly contexts that link to learners' lived experiences.
- Why do we use contexts?
  - Motivation/relevance of mathematics;
  - The context itself affords rich mathematics to emerge (e.g., maximizing area of an animal pen);
  - Help students connect informal knowledge to mathematical representations.

We  contexts too!

But ... our relationship with contexts is complicated.

What have we learned?

Findings

- *The Money Problem*. Percentage correct and most popular student-generated equations

Grade	% Correct	8 + 5 =13	-8 + -5 = -13	-8 - 5 = -13	Other
2nd/4th w/o negatives	92.5%	85%	0%	0%	15%
2nd/4th w/ negatives	97.5%	95%	2.5%	0%	2.5%
7th	100%	82.5%	10%	5%	2.5%
11th	100%	65%	17.5%	12.5%	5%

- 77.5% of 7th graders and 47.5% of 11th graders generated no equation with negatives.

- When presented with equations involving negative numbers (e.g., -8 + -5 = -13), about half of 7th graders and 60% of 11th graders interpreted -8 as meaning the borrower has a debt of 8 dollars; about half of 7th graders and 35% of 11th graders interpreted -8 as as loss of $8 from the lender' s perspective.

Video

- Lindsey, Gr 11

Video

- Ciara, Gr 11

What did we learn from the money problem?

- More than 90% of students without negative numbers were able to successfully solve the money problem.
- More than half of students with formal, school-based instruction on integers did not show evidence of initially seeing the money problem as involving negative integers.
- When presented with negative number equations, 7th and 11th graders had different interpretations of what negatives meant (a loss or a debt).
- Is this context about negative numbers? For whom? And in what ways?

The Bird and Fish Problem

Findings

Bird and Fish Problem. Percentage correct and most popular student-generated equations

Grade	% Correct	20 + 5 =25 Correct	20 - -5 = 25 Correct	20 + -5 =15 Incorrect	20 - 5 =15 Incorrect	Other
2nd/4th w/ o negatives		47.5%	0%	0%	30%	22.5%
2nd/4th w/ negatives	70%	62.5%	0%	0%	35%	2.5%
7th	85%				10%	7.5%
11th	100%				0%	5%

Video

- Dillon, Gr 11

What did we learn from the bird and fish problem?

- More than half of 7th and 11th graders generated no equation with negative numbers.
- Only 20% of 7th graders and 37.5 % of 11th graders could eventually write a CORRECT negative-number sentence.
- Yet 100% of 11th graders got the problem correct.
- When presented with the equations 20 – -5 and 20 + -5, 7th and 11th graders had difficulty reasoning about the meanings of the operations. Few students interpreted subtraction as a difference.
- Is this context about negative numbers? For whom? In what ways?

Take Aways ...

- ~~Don't use contexts~~ **NOT!**
- When you use contexts, start where the students are. Recognize that when you use contexts, students may think about them differently than you do.
- We do not know what mathematical goals are supported by introducing a particular context until we know how the context is interpreted by students.

Michelle's Thoughts

Vicki's Thoughts

Discussion Questions

- What does *solve story problems with integers* mean?
- How do number sentences relate to integer reasoning?
- What supports allow us to build on students' thinking about context?
- Theoretically, what is a context?
- Are there contexts that necessitate the use of integers?