Problem Types That Evoke Particular Ways of Reasoning for Some Students

Sample Problems	Characteristics of Problems	Way of Reasoning Evoked	Description of Way of Reasoning	Create Your Own Problems
-2 + □ = 4 3 – □ = 9	• Problems "Cross Zero." • No Double Signs (No addition of a negative or subtraction of a positive)	Order-based	Responses that leverage the sequential and ordered nature of numbers to reason about the problem. Strategies may include use of the number line with a motion metaphor, counting forward or backward by ones, or jumping to zero.	
-5 + -1 = □ -7 – □ = -5	• All numbers in the problem are negative.	Analogically based (e.g., negatives like positives)	Responses supported by comparison to other entities that the student deems to be, in some way, structurally similar to negative integers. For example, the student may compare negative integers to positive integers, negative charges, or dollars owed.	
6 + □ = 4 5 – □ = 8 6 – 2 = □ vs. 6 – -2 = □ -5 + -1 = □ vs. -5 – -1 = □	• Problems that contradict "addition makes bigger" or "subtraction makes smaller" • Pairs of problems with exactly one feature changed.	Formal Mathematical	Responses that leverage generalized principles and properties of mathematics. Strategies may include *inferring the sign* (i.e., determining the sign of the answer before determining the magnitude) on the basis of features of the problem or using *logical necessity* (i.e., generalizing beyond a specific case by making a comparison to another known problem and adjusting one's heuristic so that the logic of the approach remains consistent).	
6 – -2 = □ 5 – □ = 8	• Problems with double signs	Computational	Responses driven by the use of a rule or computation to arrive at one's answer.	

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