Opportunities for Algebraic Reasoning in the Context of Integers

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Project **Z**: Mapping Developmental Trajectories of Students' Conceptions of Integers

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Algebraic reasoning?

Liberty, Grade 2

$-3 + 6 = \square$ & $-8 - 3 = \square$

The intersection of algebra and integers

- Integer arithmetic is difficult for students as is solving algebraic equations involving integers (Kloosterman, 2012 & in press; Vlassis, 2002, 2008).
- Why?
- The abstract nature of integers--it is hard to conceive of numbers as having magnitude when they are negative or less than zero.
- 1, 2, 3, ... are called *natural* numbers for a reason.

The intersection of algebra and integers

- Understanding and operating with integers is an important transition BECAUSE of the abstract and formal reasoning they (may) require.
- Important algebraic ideas such as additive inverses, negation, meaning of variables, and symbolic flexibility are formally addressed within the topic of integers

Logical necessity as a form of algebraic reasoning

- using and identifying underlying structures (of W) that are abstracted and generalized from computations (following Kaput, 1998)
- maintenance of consistency with what one knows to be true for existing number systems

Project Z study design

- Our work is grounded in a children's mathematical thinking perspective.
- Research questions: What are K–12 students' conceptions of integers, and what are possible learning trajectories?
- conducted 160 problem-solving interviews
  - 40 students from each of grades 2, 4, 7 & 11 across eleven ethnically diverse school sites
  - majority of questions were open number sentences (e.g., 3 – 5 = __ or -3 + __ = 6)
  - share findings from 2 codes related to algebraic reasoning: logical necessity & nonequivalent transformations

Logical necessity

Logical necessity

the case of Ryan, Grade 1

Logical necessity

the case of Beth, Grade 11

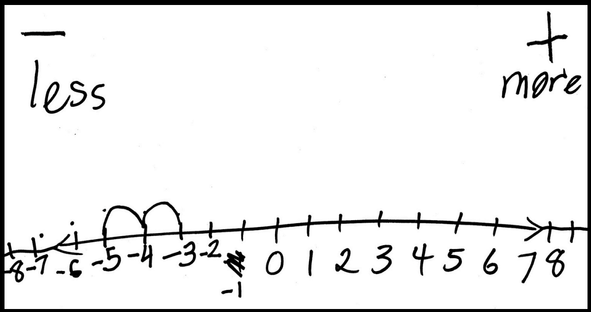
Logical necessity

the case of Violet, Grade 2

- Violet, a second grader, will use a number line and the idea of motion to try to solve each of the following problems:
    - ___ + 5 = 3
    - -5 – 4 = ___
    - 5 + ___ = 2
    - 3 – ___ = -2
- She correctly solves 3 of the 4. Predict which one she did not solve correctly.

Logical necessity

the case of Violet, Grade 2



What next?

- Think about Violet ...
  - She solved ___+ 5 =3, 3 – ___ = -2, and -5 – 4 = ___ on the number line but could not solve 5 + ____ = 2 and insisted that solving 6 + -2 is impossible.
  - Why did she struggle to solve 5 + ___ = 2 and 6 + -2 = ____?
  - What problem(s) might you pose next to extend her thinking?

Logical necessity

the case of Violet, Grade 2

Nonequivalent transformations

Solve, then predict

6 – -2 = __ 5 – __ = 8

- How do you think about these problems?
- How might middle and high school students solve these two problems?
- Do you know what *kcc* stands for?

Rule Following vs. Sense Making

Nonequivalent transformations

the case of Valentina (and friends), Grade 7

As you saw earlier, one problem we posed was
6 – -2 = ___.

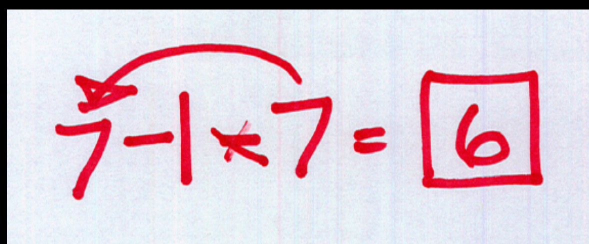
If a student solved 6 – -2 = __ by changing it to 6 + + 2, the interviewer asked, “Before you changed the problem, was the answer to the original problem 8?”

Nonequivalent transformations

the case of Al, Grade 11

Nonequivalent transformations

the case of Callie, Grade 7



Data

Table 1. Comparison of Frequency Counts Across Codes

	Logical Necessity			**Nonequivalent Transformation**		
	Number of students	Number of occurrences	Number of distinct problems	Number of students	Number of occurrences	Number of distinct problems
Grades 2 & 4	4	7	6	17	41	9
Grade 7	6	9	7	9	30	19
Grade 11	7	15	12	3	7	7

Questions

- Why do we see more instances of nonequivalent transformations than logical necessity?
- For whom is 6 – -2 equivalent to 6 + +2 and in what ways?
- Why are some children able to reason algebraically about integers?
- What implications can we draw regarding integer instruction?
- What kinds of tasks should we use for integer instruction?
- What is the goal of integer instruction?

Thank you

Algebraic Reasoning

FIVE FORMS OF ALGEBRAIC REASONING

In my view, algebraic reasoning is a complex composite of five interrelated forms of reasoning. The first two underlie all the others (kernels), the next two constitute topic strands, and the last reflects algebra as a web of languages. All five richly interact conceptually as well as in activity—to understand *this* algebra is to make connections. All five can and should be started early.

1. Algebra as Generalizing and Formalizing Patterns and Constraints, especially, but not exclusively, Algebra as Generalized Arithmetic Reasoning and Algebra as Generalized Quantitative Reasoning;
2. Algebra as Syntactically-Guided Manipulation of Formalisms;
3. Algebra as the Study of Structures and Systems Abstracted from Computations and Relations;
4. Algebra as the Study of Functions, Relations, and Joint Variation; and
5. Algebra as a Cluster of (a) Modeling and (b) Phenomena-Controlling Languages.