LECTURE NOTES – PHYSICS 564 NUCLEAR PHYSICS

While this outlines many of the key concepts, it should not replace lecture notes.

Units:
Length: 1 ångstrom = 10^{-10} m = 1 Å
1 fermi (or femtometer) = 10^{-15} m = 1 fm.
Energy: 1 electron volt (eV) = energy of electron accelerated through 1 volt electrical potential = 1.6 \times 10^{-19} J.
1 keV = 10^3 eV; 1 MeV = 10^6 eV; 1 GeV = 10^9 eV.
1 u (atomic mass unit, sometimes also a.m.u.) = 931.5 MeV (where ^{12}\text{C} has mass = 12.00000)

m_p = 938.3 \text{ MeV} \quad m_n = 939.6 \text{ MeV} \quad m_e = 511 \text{ keV}
\hbar c = 197.3 \text{ MeV-fm}.

Basic definitions
A chemical element denoted by # of electrons = # of protons.
Z = # of protons, N = # of neutrons.
A=Z+N = mass number.
nuclide = a nuclear species, denotes by Z, N. (e.g. ^{14}\text{C})
isotope = same Z, different N. (e.g. ^{12}\text{C}, ^{14}\text{C})
isotone = same N, different Z (e.g. ^{12}\text{C}, ^{14}\text{O})
isobar = same A (e.g. ^{14}\text{N}, ^{14}\text{C})
isomer = same isotope but excited state (usually long-lived) (e.g. ^{189}\text{Au}, stable; ^{189m}\text{Au}, half-life = 4 minutes) abundance = relative percentage (by number) of a nuclide/isotope.

isotopic nomenclature: \(^{A}X^{[N]}\) X = chemical species and implicitly denotes Z. Typically ^{12}\text{C} although ^{12}\text{C}_6 is possible to aid bookeeping. (Also possible is \(^{A}ZX^{N}\), such as ^{14}6\text{C}—again, only for pedantic and bookeeping purposes.)

Binding energy and reaction Q-value
binding energy is the energy released when a system of particles is bound together, or the energy required to totally dissociated a nucleus into its component protons and neutrons.
\[ \text{BE} = \left[ Zm_p + Nm_n - m(N,Z) \right] c^2; \] usually in MeV (or in some tables, keV).
Mass excess or mass defect : \( \Delta = (m(N,Z) - A) \times 931.5 \text{ MeV/u} \) where m(N,Z) is in u and A=N+Z is mass number, so that \( \Delta \) is in MeV.
packing fraction = -BE/A which related how tightly bound/particle the system is.
Is approximately -8 MeV / nucleon for most nuclides.

reaction Q-value: In a reaction \( A \rightarrow B \) (where A, B can be systems of particles), Q is the energy “released”, e.g. \( A \rightarrow B + Q \). Q can be negative.
If Q > 0, reaction is “exothermic” (releases energy); reaction can proceed without additional energy.
If Q < 0, reaction is “endothermic” and requires energy to proceed.
separation energy = energy required to separate a particle or particle,
e.g., neutron separation energy \( S_n = (m(A^{-1}X_{N-1})+m(n) - m(A^{X}N)) c^2 \)
\[ = BE(A^{X}N) - BE(A^{-1}X_{N-1}) \]
Nuclear Radii

Assume *spherically symmetric* distributions \( \rho(r) \) (not necessarily true!)

*rms (root-mean-square) radius* \( R_{\text{rms}} \)

\[
R_{\text{rms}} = \left\langle r^2 \right\rangle^{1/2}; \quad R_{\text{rms}}^2 = \frac{\int r^2 \rho(r) d^3r}{\int \rho(r) d^3r}
\]

Systematics: empirically one finds \( R_{\text{rms}} \approx r_0 A^{1/3} \), where \( r_0 \approx 1.2-1.4 \text{ fm} \).

(This implies approximately constant density and is a consequence of the short-range nature of the nuclear force.)

For a sphere of constant density out to radius \( R_{\text{max}} \), one finds \( R_{\text{rms}} = \frac{1}{2} R_{\text{max}} \).

One must be careful to distinguish between the matter density and the charge density. For a sphere of uniform charge density out to \( R_{\text{max}} \), the electrostatic or Coulomb energy is

\[
E_C = \frac{3}{5} \frac{ke^2 Z^2}{R_{\text{max}}}, \quad \text{where} \quad ke^2 = 1.44 \text{ MeV-fm}.
\]

*Mirror nuclei* are isobars (same \( A \)) with opposite numbers of protons and neutrons. For example, \(^{14}\text{C} (Z=6, N=8)\) and \(^{14}\text{O} (Z=8, N=6)\) are mirror nuclei.

Scattering

*Flux* = # of particles / unit area / time

*cross-section* has units of area. To get the reaction rate per target particle, multiply flux x cross-section = reactions / time.

A standard unit in nuclear physics is the *barn* (b) = \( 10^{-24} \text{cm}^2 = 100 \text{ fm}^2 \).

One frequently uses the millibarn (mb) = \( 10^{-3} \text{ b} \) and *nanobarn* (nb) = \( 10^{-9} \text{ b} \).

*Momentum transfer*: for the initial and final momenta \( k_i, k_f \), of the *projectile*, the momentum transfer \( q = k_i - k_f \).

In general scattering cross-section \( \sigma_{\text{scat}} = \sigma_{\text{Rutherford}} \times |F(q)|^2 \), where Rutherford scattering is the scattering from a point particle and the *form-factor* \( F(q) \) is a correction due to an extensive particle. (This is true for both total and differential cross-sections.) In its simplest form, the form factor is the Fourier transform of the charge density.

For low momentum transfer, \( F(q) \approx Z(1 - 1/6 q^2 R_{\text{rms}}^2 + ...) \).

(N.B. Actually scattering theory and form factors can be more complicated than this, but for the simplest cases the above are approximately true.)

\[
F(q) = \text{Fourier transform of charge density} = \int d\vec{r} e^{i \vec{q} \cdot \vec{r}} \rho(\vec{r}) = \frac{4\pi q^2}{q} \int_0 \sin(qr) \rho(r) dr
\]
Quantum theory of angular momentum

**orbital angular momentum** (from relative motion) is quantized in units of \( \hbar \)
\[ L = 0\hbar, 1\hbar, 2\hbar, \ldots \]

**spin** (intrinsic angular momentum) can be either integral or half-integral:
- *fermions* have half-integral spin \( s = \frac{1}{2} \hbar, \frac{3}{2} \hbar, \frac{5}{2} \hbar, \ldots \)
- *bosons* have integral spin \( s = 0\hbar, 1\hbar, 2\hbar, \ldots \) (usually drop the \( \hbar \)).

Typical fermions include electrons, protons, neutrons, quarks, neutrinos...

Typical bosons include pions, photons, W- and Z-bosons, gluons, (gravitons).

Components: Classically, angular momentum \( J \) has 3 components, \( \mathbf{J} = (J_x, J_y, J_z) \).
In QM we can only discuss the total angular momentum \( J \) and one component, usually \( J_z \).
(The other components are indeterminant). \( J_z \) can take on the values \( J_z = -J, -J+1, -J+2, \ldots J-1, J \).
Sometimes for \( J_z \) we write \( m \). So the angular momentum for a particle (or system of particles) is denoted by \( (J, J_z) \) or \( (J, m) \)

Adding angular momentum: The Rules
Suppose we start with \((J_1, m_1)\) and \((J_2, m_2)\) and “add” them together. What is final \((J, m)\)?
(1) \( z \)-component is added: \( m = m_1 + m_2 \).
(2) \( |J_1-J_2| \leq J \leq J_1+J_2 \). The result is in fact a linear superposition of those possible \( J \)’s, with the probability amplitudes given by *Clebsch-Gordon coefficients* (which we will ignore as much as possible).

Parity
Wavefunctions \( \Psi \) such that \( \Psi(-\mathbf{r}) = \Psi(\mathbf{r}) \) have even parity; wavefunctions such that \( \Psi(-\mathbf{r}) = -\Psi(\mathbf{r}) \) have odd parity. Parity is a multiplicative quantum number and usually denoted by \( \pi \). Electromagnetism and the strong nuclear force conserve parity, but the weak nuclear force does not conserve parity. For example, neutrinos are “left-handed” (which means their spins are anti-aligned with their momenta).

Quantum statistics of indistinguishable particles
A wavefunction of two or more indistinguishable particles (e.g. all electrons or all protons) is either symmetric or antisymmetric under interchange of the particles’ identities:
- \( \Psi(\mathbf{r}_1, \mathbf{r}_2) = + \Psi(\mathbf{r}_2, \mathbf{r}_1) \) symmetric (for *bosons*)
- \( \Psi(\mathbf{r}_1, \mathbf{r}_2) = - \Psi(\mathbf{r}_2, \mathbf{r}_1) \) antisymmetric (for *fermions*). This leads to the *Pauli exclusion principle*: no two identical fermions can be in exactly the same quantum mechanical state.
Isospin
Protons and neutrons have *intrinsic spin* $= \frac{1}{2}$, and can have 3rd component $s_z = \frac{1}{2}$.
In analogy to this, one can treat protons and neutrons as identical particles, both with isospin $= \frac{1}{2}$ . Total isospin is represented by either $T$ or $I$. Protons and neutrons have different $T_z$. I will use protons have $T_z = + \frac{1}{2}$ and neutrons $T_z = - \frac{1}{2}$ (note sometimes you will see it the other way). The coulomb interaction does not conserve isospin—since protons have charge +1 and neutron charge 0, but the strong nuclear force does conserve isospin; hence isospin is a good approximate quantum number for nuclei.

Radioactive Decay
The fundamental decay laws. Let $N$ be the number of radioactive nuclei.

\[
dN/dt = -\lambda N \quad \text{or} \quad N(t) = N_0 \exp(-\lambda t).
\]

- **mean life**: $\tau = 1/\lambda$

- **half-life**: time in which half of all nuclei decay away $t_{1/2} = \tau \ln 2 = \ln 2 / \lambda$.

1 Curie (Ci) = $3.7 \times 10^{10}$ decays/s
1 Becquerel (Bq) = 1 decay/s

---

RADIOACTIVE DECAY MODES

*decay*: $X \rightarrow A+B+..$

*reaction*: $X+Y \rightarrow A+B+..$

A decay mode is possible only if the Q-value for the reaction is positive.

*a particle is stable* if no possible decay modes have positive Q-values.

*branching ratio* is the percentage of time a particular decay mode occurs.

**ALPHA (α) DECAY**

α-particle is nucleus of $^4$He.

$^A_XN \rightarrow ^A^{Z-2}X_{N-2}+\alpha$.

-governed by tunneling through Coulomb barrier. Limited to heavy ($A \geq 146$) nuclei.

**BETA (β) DECAY**

- **β⁻** electron (e⁻); **β⁺** positron (e⁺) or EC electron capture

- **β⁻ decay**: $^A_ZX \rightarrow ^{A-1}_{Z+1}Y+e^-+\bar{\nu}_e$

- **β⁺ decay**: $^A_ZX \rightarrow ^{A+1}_{Z-1}W+e^++\nu_e$

- **EC**: $^A_ZX+e^- \rightarrow ^{A-1}_{Z-1}W+\nu_e$
Beta-decay, continued

NOTES: the ‘daughter’ products of both \( \beta^- \)-decay and EC are the same.
Q-value of EC is 1.022 MeV higher than that of \( \beta^- \)-decay so more common.
Beta-decay governed by weak interaction.

*Discovery of the neutrino*
Existence of neutrino implied by applying two conservation laws to beta-decay:
conservation of angular momentum (a spin = \( \frac{1}{2} \) particle ought not to decay to 2 spin \( \frac{1}{2} \) particles but requires a 3rd) and energy (electron energy is continuous spectrum up to Q-value; neutrino must carry away the remaining energy).

*Lepton number* is an additive quantum number that is conserved in all reactions.
Electrons and neutrinos have lepton number +1, positron (anti-electrons) and anti-neutrinos have lepton number −1.

**GAMMA (\( \gamma \)) DECAY**

Nucleus in an excited state decays to ground state by emitting a high-energy photon (\( \gamma \)), usually a few hundred keV or a few MeV.

\[ _{A}^{X^*} \rightarrow _{A}^{X} + \gamma \]
a common by-product of other decays (e.g., beta- and alpha-decay).

**NEUTRONS**
normal, non-ionizing; not a normal decay product but can be captured and induce gamma-emission.
-can be produced in fission, can induce fission.

**FISSION**
Heavy nucleus breaks into large fragments. Spontaneous rare for \( A < 250 \); can be induced by neutron capture and release additional neutrons in the process.

**SHIELDING**
Alpha rays can be shielded by a piece of paper
Beta rays by a thin sheet of metal.
Gammas require thicker metal, often of high Z such as lead.
Neutrons moderated (slowed down) by low Z materials, captured by boron, cadmium.
Detecting radiation
Detect ionizing radiation (non-ionizing radiation such as neutrons can only be detected secondhand, by capture reactions that release ionizing radiation such as γ’s). Ionizing radiation can be identified by

\[
\text{range} = \text{distance particle of given energy travels before stopping}
\]

or

\[
\text{stopping power} = \text{loss of kinetic energy per unit length } dE/dE. \text{ Range is integral of } (dE/dx)^{-1}.
\]

* “Heavy” particles, including protons, lose energy through collisional loss with electrons.
* Electrons lose KE via both collisions and bremsstrahlung (“braking radiation” = radiation emitted from acceleration/deceleration).
* Photons (gamma rays) lose energy through: photoelectric absorption on electrons, Compton scattering off electrons, and e⁺e⁻ pair production.

Ionizing radiation can also be identified by turning radius in magnetic field, and by decay modes.

Detectors
We discussed three main classes of detectors: gas counters, semiconductors, and scintillation counters. In gas counters and semiconductors, ionized electrons directly provide a current that is amplified into a signal. In scintillation counters, ionized electrons converted into light that is then transmitted to phototube (where the scintillation light produces more electrons and eventually a current).

**energy resolution**: no detector can determine absolutely the energy of an ionizing particle. The uncertainty or distribution of the energy for a monoenergetic source is the energy resolution.

**efficiency**: no detector detects all particles that pass through. The fraction of particles passing through a detector that are actually detected is the efficiency.

Accelerators
In all accelerators, electric fields provide acceleration and magnetic fields guidance and focussing.

* Electrostatic accelerators, such as Crockcroft-Walton and van de Graf, provide a high electric field for acceleration. Limit of energy is a few MeV, limited by the electrostatic breakdown of materials.
* Cyclotrons have two electrodes, or dees, in a constant magnetic field. By cycling the charge on the dees at the cyclotron frequency \( \omega_{\text{cyc}} = qB/2\pi m \) one can repeatedly accelerate a particle by the same electric field. Limited by needing a large magnetic field over a large area.
* Synchrotrons are similar to cyclotrons but limit path of particles to an evacuated tube of a fixed radius; thus both magnetic field and acceleration frequency must be varied in synchrony. Most very large accelerators are synchrotrons.
* Linear accelerators are used for electron accelerators, to avoid loss of energy via bremsstrahlung.
* Colliders are most efficient in providing large center-of-mass energy of collisions.
Guidance by magnetic fields

*Dipole* magnets are used to steer the beam.

*Quadrupole* magnets are used to focus the beam. Note that two sets of quadrupoles are necessary.

Nuclear binding energies

*mass defect* $\Delta = (m - A \ u)c^2$ in MeV; here $m$ is also in atomic mass units, so must use conversion 931.5 MeV/u

*semi-empirical formula for binding energies* (Bethe, Weizsaecker 1935, 1936)

$$BE = a_{\text{vol}} A + a_{\text{surf}} A^{2/3} + a_{\text{coul}} Z(Z-1) A^{-1/3} + a_{\text{sym}} (N-Z)^2/A + \delta(A)$$

**typical values** (from Krane, p 68)

- *volume energy* $a_{\text{vol}} = +15.5$ MeV
- *surface energy* $a_{\text{surf}} = -16.8$ MeV
- *Coulomb* $a_{\text{coul}} = -0.72$ MeV
- *symmetry* $a_{\text{sym}} = -23$ MeV
- *pairing* $\delta(A) = 34 A^{-3/4}$ MeV for $Z$ even, $N$ even
  - $-34 A^{-3/4}$ MeV for $Z$ odd, $N$ odd
  - $0$ for $A$ odd.
Microscopic picture of nuclear forces

Modern picture of interactions: all "forces" mediated by the exchange of particles: photons are the exchange particle of the electromagnetic force.

For nuclear physics, it is sufficient to consider specific mesons (generally subatomic particles intermediate in mass between leptons, such as electrons and neutrinos, and baryons, such as protons and neutrons) as the carrier of the nuclear force. In particular the pi meson or pion (mass about 140 MeV) and the rho and omega mesons (masses around 770 MeV) as the main carriers of the nuclear force. The symbols are \( \pi, \rho, \omega \).

Yukawa interaction. The Yukawa potential is a generalization of the familiar \( 1/r \) potential:

\[
V_{\text{Yukawa}}(r) = \frac{\exp(-mc^2 r / \hbar c)}{r},
\]

where \( m \) is the mass of the exchange particle. Note that as the mass goes to zero, we regain the usual \( 1/r \) potential. The range of the interaction is given by \( \hbar c / mc^2 \). Thus the range of pion exchange, for example, is about 1.4 fm.

The interaction between nucleons is "long" range, due to pions, is attractive, but at very short range it turns to repulsive due to the exchange of rho and omega mesons.

Nuclear structure

The simplest model of nuclear structure is the liquid drop model, where one pictures the nucleus as a droplet of nuclear fluid. When quantized, as done by A. Bohr and B. Mottelson, it describes much of the "typical" behavior of nuclear excitation spectra.

All even-\( Z \), even-\( N \) nuclei have \( J=0 \) ground states. The low-lying spectra often come in a sequence then of \( J=2,4,6,... \)

If the excitation energy \( E(J) = E_0 + \text{const} \times J \), then it is a vibrational spectrum. This is just like a harmonic oscillator. We picture the ground state as being spherical and having small vibrations about the spherical shape.

If the excitation energy \( E(J) = E_0 + \text{const} \times J(J+1) \), then it is a rotational spectrum. The ground state is deformed and can rotate about an axis. If the deformation is longest along the remaining symmetry axis (cigar-shaped) it is prolate; otherwise it is oblate (pancake-shaped).

Quantum mechanical expression for rotational KE = \( \frac{1}{2} \hbar^2 J(J+1) / I \), where \( I \) is the moment of inertia. (This is in fact a semi-classical expression and not fully Q.M.)
The Shell Model and the Independent Particle Picture

Fully quantum mechanical picture of many-body system: find $N$-body wavefunction $\Psi(r_1, r_2, r_3, r_4, \ldots r_N)$ by solving the $3N$-dimensional Schrodinger eqn—too hard!

Instead approximate with a one-body Schrodinger eqn:

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{mean-field}}(r) \right) \psi(\vec{r}) = \varepsilon \psi(\vec{r}) ; \text{ Here } V_{\text{mean-field}}(r) \text{ averages the interaction with all the other particles.}$$

This can be (exactly) reduced to a one-dimensional radial Schrodinger eqn:

let $\psi(r, \theta, \phi) = u(r) / r \; Y_{lm}(\theta, \phi)$, where $Y_{lm}(\theta, \phi)$ is a spherical harmonic function with orbital angular moment $l$ and $z$-component $m$.

The radial Schrodinger eqn is

$$\left( -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l + 1)}{2mr^2} + V_{\text{mean-field}}(r) \right) u(r) = \varepsilon u(r) ; \text{ note this depends on } l \text{ but not } m.$$ 

Solving the radial Schrodinger equation—which we will thankfully leave to someone else!!—yields a set of single-particle energies and single-particle states that define the shell structure.

Single-particle states

We label single-particle states by 3 quantum numbers:

- $l$, the orbital angular momentum
- $j$, the total angular momentum $= l \pm \frac{1}{2}$ (since must add spin $s = \frac{1}{2}$)
- $n$, the nodal quantum number $= 0, 1, 2, 3...$

We also can include label states by $m$ (or $j_z$) the 3rd component of ang. momentum.

Single particle states are degenerate if they have the same single-particle energy. A state with some given $j$ can have $j_z = -j, \ldots, +j$, so a degeneracy of $(2j+1)$. In addition, states with different $j$, or $l$, or $n$, can sometime be degenerate for certain potentials. (NB: this is usual because of a symmetry, but we won’t worry about that.)

Spectroscopic notation

use letters to denote orbital ang momentum:

$l = 0 \; s; \; l=1 \; p; \; l=2 \; d; \; l=3 \; f; \; l=4 \; g,$ and it continues alphabetically.

states labeled by $n(l)_j$ : hence $0s_{1/2}, \; 0p_{3/2}, \; 1s_{1/2}$, etc.

If we ignore spin, then just n l.
Examples:

**Coulomb:**
0s lowest shell
1s, 0p degenerate
2s, 1p, 0d degenerate
3s, 2p, 1d, 0g degenerate...

In actual atoms, higher \( l \) orbitals pushed up in energy due to shielding from inner electrons.

Note, we can understand this through the *principal quantum number* \( N = 1, 2, 3, ... \)
The energy of bound states go as \( 1/N \), hence "principal." For the Coulomb potential *only*, we have \( N = n + l + 1 \). Thus for \( N = 1 \), we have only have \( n = 0, l = 0 \). For \( N = 2 \) we can have \( n = 0, l = 1 \) and \( n = 1, l = 0 \), and so on. Also note that in atomic physics the convention is to label by \( N(l) \), so \( N = 1 \) has the 1S, \( N = 2 \) has the 2S and 2P, \( N = 3 \) has 3S, 3P, 3D, and so on.

**Harmonic Oscillator:**
0\( \hbar \omega \): 0s
1\( \hbar \omega \): 0p
2\( \hbar \omega \): 1s, 0d
3\( \hbar \omega \): 1p, 0f
4\( \hbar \omega \): 2s, 1d, 0g

Here the energy is \( \hbar \omega (N + 1/2) \) where again \( N \) is the principal quantum number, with \( N = 0, 1, 2, 3, ... \). But unlike Coulomb, for the 3-D harmonic oscillator, \( N = 2n + l \). Hence for \( N = 0 \) we can only have \( n = 0, l = 0 \) (0s), for \( N = 1 \) we can only have \( n = 0, l = 1 \) (0p), for \( N = 2 \) we have both \( n = 0, l = 2 \) (0d) and \( n = 1, l = 0 \) (1s), for \( N = 3 \) we have 0f and 1p, etc.

**Nuclei**
Nuclear single-particle level experience *spin-orbit splitting*. That is, states of the same \( l \) but different total \( j \) are widely separated in energy, for example 0p\(_{1/2}\) and 0p\(_{3/2}\). (In atoms there is only a very small difference; this is known as *fine structure* there.)

Nuclear single-particle orbits are grouped into “major shells” that track harmonic oscillator levels but with significant differences, notably spin-orbit splitting. The order of levels are (in order of being filled, that is, lowest energy first:)

0s\(_{1/2}\)

--------- [2]
0p\(_{3/2}\)
0p\(_{1/2}\)

--------- [8]
0d\(_{5/2}\)
1s\(_{1/2}\)
0d\(_{3/2}\)

--------- [20] the numbers in brackets denote how many particles are filled at the major jumps in single-particle energies;

0f\(_{7/2}\)
These are magic numbers:

\[ 1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2} \]

--- [50]

\[ 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2} \]

--- [82]

\[ 0h_{9/2} \]

**Parity**: Parity for a given orbital angular momentum. The parity is \((-1)^l\); hence \(s,d,g\) have positive parity and \(p,f,h...\) have negative parity. Parity is multiplicative, meaning that a nucleus with two negative parity single-particle states will have overall positive parity.

**Rules for guessing ground state** \(J^p\) **from single-particle states**:

1. All even-\(N\), even-\(Z\) ground states are \(0^+\).
2. For odd \(A\), look at the last particle or "hole" that is filled, using the order described above. The nucleus will have \(J^p\) of that single-particle state. (The other neutrons and protons will tend to pair to \(J=0\)).
3. Odd-\(N\), odd-\(Z\) are generally not simply predictable (and also rare).
Microscopic theories of transitions

“multipolarity” of a transition \( J_i^\pi \rightarrow J_f^\pi \) is \( \Delta J^\pi \) where 
\[
| J_i - J_f | \leq \Delta J \leq J_i + J_f \quad \text{and} \quad \Delta \pi = \pi_i \pi_f
\]

Beta-decay

Classification of decays:
\( \Delta J^\pi = 0^+ \): “Fermi” transition
\( \Delta J^\pi = 1^+ \): “Gamow-Teller” transition
all others are classified as “forbidden.”

Total decay rate and \( f_{1/2} \) values

decay rate \( \lambda = \frac{G_F^2 | M_\beta |^2 m_e^5 c^4}{2\pi^3 h^7} f(Z, Q) \),
where \( G_F = 1.0 \times 10^{-5} / (m_{\text{nucleon}})^2 \) is the Fermi constant, 
\( M \) is the nuclear beta-decay matrix element, (contains information about allowed or forbidden), and \( f(Z, Q) \) is the (dimensionless) Fermi integral, which contains all the kinematics.

Introduce \( f_{1/2} = \frac{2\pi^3 h^7 \ln 2}{G_F^2 | M_\beta |^2 m_e^5 c^4} \) (always in seconds); often use \( \log f_{1/2} \).

The smaller the \( f_{1/2} \)-value, the faster the decay (for a given \( Q \)).
Smallest possible \( f_{1/2} = 3100 \) sec = “superallowed” (log \( f_{1/2} =3.5 \))
Typical \( f_{1/2} \) for allowed transitions is \( 10^5 \) (so log \( f_{1/2} = 5 \)).
Typical \( f_{1/2} \) for forbidden transitions is \( 10^{6-9} \) (so log \( f_{1/2} = 6-9 \)) but can be larger.

Sargent’s rule
For similar systems (similar \( A, Z \), similar log \( f_{1/2} \) values)
decay rate \( \lambda \propto Q^5 \).

So \( \lambda_i \propto Q^5 \times 10^{-(log f)} \) for a given transition.

Branching ratio for transition \( i \) is \( \lambda_i / \lambda \) where \( \lambda = \sum \lambda_i \)
Gamma decay

Labeling of transitions by $\Delta I$ of transition:
if $\pi = (-1)^{\Delta I}$, then E$^{+J}$ transition (electric): $E1 = 1^+$, $E2 = 2^+$, $E3 = 3^+$, etc.
if $\pi = -(-1)^{\Delta I}$, then M$^{+J}$ transition (magnetic): $M1 = 1^+$, $M2 = 2^+$, $M3 = 3^+$, etc.

A single photon cannot be emitted in $0^+$ or $0^-$ transitions; either emit two photons or (most common) eject an inner electron; the latter is called internal transition (IC).

Weisskopf estimates

$$\lambda (E1) = 1.0 \times 10^{14} \ A^{2/3} (E_\gamma)^3$$
$$\lambda (E2) = 7.3 \times 10^7 \ A^{4/3} (E_\gamma)^5$$
$$\lambda (E3) = 34 \ A^2 (E_\gamma)^7$$
$$\lambda (E4) = 1.1 \times 10^{-5} \ A^{8/3} (E_\gamma)^9$$

$$\lambda (M1) = 5.6 \times 10^{13} \ (E_\gamma)^3$$
$$\lambda (M2) = 3.5 \times 10^7 \ A^{2/3} (E_\gamma)^5$$
$$\lambda (M3) = 16.0 \ A^{4/3} (E_\gamma)^7$$
$$\lambda (M4) = 4.5 \times 10^{-6} \ A^2 (E_\gamma)^9$$

where $\lambda$ is in s$^{-1}$ and $E_\gamma$ is in MeV.

To compute the actual strength of a transition (from experimental half-life) in Weisskopf units,
$$\lambda_{\text{expt}} = \ln 2 / t_{1/2}$$

$$\text{strength} = \lambda_{\text{expt}} / \lambda_{\text{Weisskopf}} \ (\text{in w.u.})$$

a transition whose strength is much larger than 1 w.u. is labeled collective; otherwise it is noncollective.
Introduction to particle physics

**FORCES**

strong nuclear force: interacts only with quarks. exchange of *gluons* (g)
  quantum theory: QCD (quantum chromodynamics)
electromagnetic force: interacts with quarks + charged leptons: exchange of photons (γ)
  quantum theory: QED (quantum electrodynamics)
weak nuclear force: interacts with quarks + leptons: exchange of charged $W^\pm$ bosons,
  neutral $Z^0$ bosons.
  quantum theory: (combined with QED) Weinberg-Salam (-Glashow) (WS theory) or *electroweak theory*.

gravity: interacts with all matter and particles; exchange of gravitons?
  no empirically tested quantum theory; perhaps superstrings...?

*Grand Unified Theories* (GUT) seek to unify QCD with the Weinberg-Salam theory of electroweak interactions. Predicts proton decay (not yet seen).

*Theories of Everything* (TOE) include gravity; much more speculative.

**MATTER:**

note: I am not including all possible decays, only a few

Leptons

All leptons have spin = $\frac{1}{2}$

3 generations:

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Charge</th>
<th>Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-$ electron</td>
<td>0.511</td>
<td>$-1$</td>
<td>stable</td>
</tr>
<tr>
<td>$\nu_e$ electron neutrino</td>
<td>0</td>
<td>0</td>
<td>stable</td>
</tr>
<tr>
<td>$\mu^-$ muon</td>
<td>105</td>
<td>$-1$</td>
<td>mean lifetime = $2.2 \times 10^{-6}$ s</td>
</tr>
<tr>
<td>$\nu_\mu$ muon neutrino</td>
<td>0</td>
<td>0</td>
<td>stable?</td>
</tr>
<tr>
<td>$\tau^-$ tauon</td>
<td>1785</td>
<td>$-1$</td>
<td>mean lifetime = $3 \times 10^{-13}$ s</td>
</tr>
<tr>
<td>$\nu_\tau$ tauon neutrino</td>
<td>0</td>
<td>0</td>
<td>stable?</td>
</tr>
</tbody>
</table>
total lepton number = # of charged leptons + # of neutrinos - # charged antileptons - # antineutrinos is “always” conserved. (Keep in mind that all such absolute statements are empirical. I shall not keep pointing this out.)

lepton flavor number, e.g. # electrons + # electron neutrinos - # positrons - # electron antineutrinos, is generally conserved. The charged-current weak interaction changes charged leptons into neutrinos and back and forth, usually electrons into electron neutrinos etc. If neutrinos have nonzero masses, then lepton flavor number will be violated; there is currently strong evidence for such violation (and hence for neutrino masses).

Quarks all quarks have spin = ½, and baryon number = 1/3 (antiquarks have baryon number −1/3)

\[
\begin{array}{ccc}
\text{d “down”} & q = -1/3 & \text{u “up”} & q = +2/3 \\
\text{s “strange”} & q = -1/3 & \text{c “charm”} & q = +2/3 \\
\text{b “bottom”} & q = -1/3 & \text{t “top”} & q = +2/3 \\
\end{array}
\]

Particles made of combinations of quarks are hadrons:

baryons (baryon number B =1) made of 3 quarks.
mesons (baryon number B =0) made of quark-antiquark pair.

Quarks also have color charge, R, G, or B. (Anti-quarks have anti-color.) A baryon or meson have total color = 0, that is RGB or R-anti-R + ... Details require group SU(3) and are beyond the scope of this course.
**Baryons** (an incomplete list!)

\[ I = \text{isospin} \]

<table>
<thead>
<tr>
<th>particle</th>
<th>charge</th>
<th>( J )</th>
<th>( I )</th>
<th>mass (MeV)</th>
<th>quark content</th>
<th>mean lifetime</th>
<th>some decay modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p ) (proton)</td>
<td>+</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>938.3</td>
<td>uud</td>
<td>stable (except in GUTs)</td>
<td></td>
</tr>
<tr>
<td>( n ) (neutron)</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>939.6</td>
<td>udd</td>
<td>900 s</td>
<td>( \rightarrow p^+e^- + \text{anti-}v_e )</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>1115</td>
<td>uds</td>
<td>2.6 \times 10^{-10} s</td>
<td>( \rightarrow p^+\pi^- )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \rightarrow n^+\pi^0 )</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>+</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>1189</td>
<td>uus</td>
<td>0.8 \times 10^{-10} s</td>
<td>( \rightarrow p^+\pi^0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \rightarrow n^+\pi^+ )</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>1192</td>
<td>uds</td>
<td>5.8 \times 10^{-20} s</td>
<td>( \rightarrow \Lambda^+\gamma )</td>
</tr>
<tr>
<td>( \Sigma^- )</td>
<td>-</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>1189</td>
<td>dds</td>
<td>1.5 \times 10^{-10} s</td>
<td>( \rightarrow n^+\pi^- )</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>1315</td>
<td>uss</td>
<td>2.9 \times 10^{-10} s</td>
<td>( \rightarrow \Lambda^+\pi^0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \rightarrow \Lambda^+\gamma )</td>
</tr>
<tr>
<td>( \Xi^- )</td>
<td>-</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>1321</td>
<td>dss</td>
<td>1.6 \times 10^{-10} s</td>
<td>( \rightarrow \Lambda^+\pi^- )</td>
</tr>
<tr>
<td>( \Delta^- )</td>
<td>-</td>
<td>3/2</td>
<td>3/2</td>
<td>1232</td>
<td>ddd</td>
<td>0.6 \times 10^{-23} s</td>
<td>( \rightarrow n^+\pi^- )</td>
</tr>
<tr>
<td>( \Delta^0 )</td>
<td>0</td>
<td>3/2</td>
<td>3/2</td>
<td>1232</td>
<td>udd</td>
<td>“</td>
<td>( \rightarrow n^+\pi^0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \rightarrow p^+\pi^- )</td>
</tr>
<tr>
<td>( \Delta^+ )</td>
<td>+</td>
<td>3/2</td>
<td>3/2</td>
<td>1232</td>
<td>uud</td>
<td>“</td>
<td>( \rightarrow n^+\pi^+ )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \rightarrow p^+\pi^0 )</td>
</tr>
<tr>
<td>( \Delta^{++} )</td>
<td>+2</td>
<td>3/2</td>
<td>3/2</td>
<td>1232</td>
<td>uuu</td>
<td>“</td>
<td>( \rightarrow p^+\pi^+ )</td>
</tr>
<tr>
<td>( \Omega^- )</td>
<td>-</td>
<td>3/2</td>
<td>3/2</td>
<td>1672</td>
<td>sss</td>
<td>0.8 \times 10^{-10} s</td>
<td>( \rightarrow \Lambda^+K^- )</td>
</tr>
</tbody>
</table>

*Note:* the decays that involve the weak interaction (that is, the change of flavor of at least one quark) have a much longer lifetime than those that do not. Check for yourself!
Meson (an incomplete list!)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Charge</th>
<th>$J$</th>
<th>$I$</th>
<th>Mass (MeV)</th>
<th>Quark Content</th>
<th>Some Decay Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$</td>
<td>-</td>
<td>$0^-$</td>
<td>1</td>
<td>139</td>
<td>$du$</td>
<td>$\rightarrow \mu^- + \text{anti-} \nu_\mu$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>0</td>
<td>$0^-$</td>
<td>1</td>
<td>135</td>
<td>$dd+uu$</td>
<td>$\rightarrow \gamma + \gamma$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>+</td>
<td>$0^+$</td>
<td>1</td>
<td>139</td>
<td>$ud$</td>
<td>$\rightarrow \mu^+ + \nu_\mu$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0</td>
<td>$0^-$</td>
<td>0</td>
<td>549</td>
<td>$dd+uu$</td>
<td>$\rightarrow \gamma + \gamma$</td>
</tr>
<tr>
<td>$\rho^-$</td>
<td>-</td>
<td>$1^-$</td>
<td>1</td>
<td>770</td>
<td>$du$</td>
<td>$\rightarrow \pi^0 + \pi^-$</td>
</tr>
<tr>
<td>$\rho^0$</td>
<td>0</td>
<td>$1^-$</td>
<td>1</td>
<td>770</td>
<td>$dd+uu$</td>
<td>$\rightarrow \pi^0 + \pi^0$</td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>+</td>
<td>$1^+$</td>
<td>1</td>
<td>770</td>
<td>$ud$</td>
<td>$\rightarrow \pi^0 + \pi^+$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0</td>
<td>$1^-$</td>
<td>0</td>
<td>782</td>
<td>$dd+uu$</td>
<td>$\rightarrow \pi^0 + \pi^- + \pi^+$</td>
</tr>
<tr>
<td>$K^0$</td>
<td>0</td>
<td>$0^-$</td>
<td>$\frac{1}{2}$</td>
<td>497</td>
<td>$ds$</td>
<td>$\rightarrow \pi^+ + \pi^-$</td>
</tr>
<tr>
<td>$K^+$</td>
<td>0</td>
<td>$0^-$</td>
<td>$\frac{1}{2}$</td>
<td>497</td>
<td>$us$</td>
<td>$\rightarrow \mu^+ + \nu_\mu$</td>
</tr>
</tbody>
</table>

NB: $q$ = antiquark