## Children’s Negative-Number Obstacles and Similarities to Mathematicians
*(Philipp & Bishop, TDG, February 14, 2014)*

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<th>Obstacle</th>
<th>Children’s responses</th>
<th>Mathematicians’ related responses</th>
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<td>The existence of quantities less than nothing and the lack of a tangible, concrete, or realistic interpretation for negative numbers <em>(Negatives rejected)</em></td>
<td>“Negative numbers aren’t really numbers because we don’t really count with them in school. And there’s no negative 1 cube (holds up a unifix cube).” A number is how you know how much something is.” <em>(Violet, second grade)</em></td>
<td>The Indian mathematician Bháscara I explained, “People do not approve of a negative absolute number;” thus, negative solutions were considered “incongruous” <em>(Colebrook, 1817, pp. 216–217)</em>. Fibonacci and Descartes did not accept negative solutions unless the result could be interpreted as something positive.</td>
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<td>“A number shows how much of something you have; … [-8] is not actually a number because it’s less than a number. … It just doesn’t really have any volume for it, like what it has.” <em>(Elena, third grade)</em></td>
<td>“Above all, he [the student] must reject the definition still sometimes given of the quantity -a, that it is less than nothing. It is astonishing that the human intellect should ever have tolerated such an absurdity as the idea of a quantity less than nothing; above all, that the notion should have outlived the belief in judicial astrology and the existence of witches, either of which is ten thousand times more possible.” <em>(De Morgan, 1902, p. 72)</em></td>
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<td>“Zero is nothing and negative is more nothing.” <em>(Rebecca, second grade)</em></td>
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<td>Removing something from nothing or removing more than one has <em>(Subtrahend &lt; Minuend)</em></td>
<td>“Three minus five is zero because you have 3 and you can’t take away 5, so take away the 3 and it leaves you with zero.” <em>(Sam, first grade)</em> When asked to solve 3 – 4 and 3 – 3, Sam answered 0 to both.</td>
<td>“I know some who cannot understand that to take four from nothing leaves nothing” <em>(Pascal, 1669/1941, p. 25)</em>. “3 – 8 is an impossibility; it requires you to take from 3 more than there is in 3, which is absurd. If such an expression as 3 – 8 should be the answer to a problem, it would denote either that there was some absurdity inherent in the problem itself, or in the manner of putting it into an equation.” <em>(De Morgan, 1902, p. 104)</em></td>
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<td>“Three minus five doesn’t make sense because three is less than five.” <em>(Nola, first grade)</em></td>
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<td>When solving 3 – 5 = ☐. Andrew (second grade) replied, “How come there’s 3 and take away 5? I don’t have enough. ‘Cuz look there’s 3 (hold up 3 fingers) and I cannot take away 5 ‘cuz there’s not enough.”</td>
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<td>Counterintuitive situations involving routine interpretations of addition and subtraction <em>(Addition cannot make smaller; Subtraction cannot make larger)</em></td>
<td>“4 + ☐ = 3 is not a real problem. It’s not true” [he crossed out the problem]. “Four minus 1 would equal 3.” <em>(Brad, first grade)</em></td>
<td>Diophantus claimed that the equation 4x + 20 = 4 was “absurd” because the 4 was less than the 20 units that were added <em>(Heath, 1964, p. 200)</em>. D’Alembert <em>(1751/2011)</em> argued that the equation x + 100 = 50 should have involved subtraction instead of addition and been written as 100 – x = 50.</td>
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<td>In response to the problem 6 + ☐ = 4, Brian (first grade) said, “What’s that plus for? Isn’t it supposed to be a minus?”</td>
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<td>In response to the problem 5 – ☐ = 8, Ryan (first grade) said, “I wouldn’t be able to do it because it would always be behind 8 if it was minus something. Because if it was minus 0 it would be 5. It [the difference] would always be behind 8.”</td>
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