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"Can you help me count these pennies?": Surfacing preschoolers' understandings of counting

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ABSTRACT

Capturing the breadth and variety of children's understanding is critical if studies of children's mathematical thinking are to inform policy and practice in early childhood education. This article presents an investigation of young children's counting. Detailed coding and analyses of assessment interviews with 476 preschoolers revealed understandings that would be overlooked by solely assessing the accuracy of their responses. In particular, many children demonstrated understandings of counting principles on a challenging task that were not captured by other, simpler tasks. We conclude that common approaches to capturing young children's mathematical understanding are likely underestimating their capabilities. This study contributes to researchers' understanding of what making sense of counting looks and sounds like for preschool age children (3–5 years), the development and relations among counting principles (one-to-one, cardinal, and patterns of the number sequence), and the affordances of challenging, open-ended tasks. We close by considering the implications of recognizing and building from what children know and can do for researchers, practitioners, and policymakers.

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The last decade has seen a surge of interest in the learning of mathematics in early childhood from both researchers and policymakers. While researchers continue to investigate the long-term effects of early mathematics learning (Bailey, Duncan, Watts, Clements, & Sarama, 2018; Claessens & Engel, 2013; Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Phillips et al., 2017; Watts, Duncan, Clements, & Sarama, 2018; Watts, Duncan, Siegler, & Davis-Kean, 2014), the importance of young children's early opportunities to learn mathematics is widely recognized (Hachey, 2013; Schoenfeld & Stipek, 2011). Effectively leveraging the opportunities provided by expanded access to early childhood education requires increased attention to the range and nature of young children's understandings, including their powerful, often surprising abilities to engage in substantive mathematics.

In this article, we explore the nuances of young children's mathematical thinking to better understand what they know and can do. Drawing from a study of 476 children (age 3–5 years) in 50 classrooms in two large urban metropolitan communities in the western United States, we present large-scale data documenting preschoolers' understanding of counting. We attend to the details of children's engagement in a range of counting tasks to purposefully consider the ways that understandings[1] are assessed and interpreted. We focus on counting not only because it often serves as the focal point of mathematics instruction in early childhood classrooms, but also for its central

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role as the foundation of children's intuitive problem-solving strategies. In highlighting the details of children's emergent use of counting principles, we hope to promote conversation around how researchers and practitioners might begin to broaden what counts as mathematical understanding in ways that are both connected to the development of consequential mathematics and responsive to children's unique ways of sense-making.

Young children's powerful mathematical ideas

Extensive research has documented the development of children's mathematical thinking in early childhood, both within the US and internationally (e.g., Clements & Sarama, 2007; Moss, Bruce, & Bobis, 2016; National Research Council, 2001; 2009; Perry & Dockett, 2008). This research demonstrates that young children are remarkably capable of engaging in and making sense of sophisticated mathematical ideas and that their informal mathematical understandings provide fertile ground to build from in the learning of mathematics during early childhood. For example, children's understanding of counting and quantity develop long before they enter school (Clements & Sarama, 2007; Fuson, 1988; Gelman & Gallistel, 1986; Ginsburg, 1989), and children as young as kindergarten naturally attempt to solve problems by modeling the action or relationships within story problems (Carpenter, 1985; Riley & Greeno, 1988). Young children's intuitive strategies are remarkably adaptive, allowing them to solve a variety of problems related to multiplication, division, and even fractions (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Empson, 1999; Kouba, 1989; Mack, 1993; Turner & Celedón-Pattichis, 2011).

Less understood are the kinds of mathematical understandings children bring with them into preschool classrooms, how to better recognize these assets, and how practitioners might be supported to draw on this knowledge to support learning (Graue, Whyte, & Delaney, 2014; Parks & Wager, 2015; Turner & Drake, 2015). Surfacing and building from preschoolers' rich, varied, and oftentimes unexpected capabilities is critical both to illuminate the supposed links between early math and later learning, and to disrupt the patterns of instruction commonly experienced in economically marginalized communities of color that often underestimate children's capabilities (Ladson-Billings, 1997; Parks & Bridges-Rhoads, 2012; Phillips, Voran, Kisker, Howes, & Whitebook, 1994; Valentino, 20187).

The conceptual underpinnings of counting

Young children's counting is one of the most extensively researched areas of development. Though often considered a basic skill, counting a set of objects[2] is a complex and interconnected process that requires the coordination of multiple mathematical ideas. Gelman and Gallistel's (1978/1986) groundbreaking series of studies drew attention to the conceptual ideas that underpin children's counting processes. They identified five principles that guide meaningful counting: 1) the *one-to-one principle* – each object in a collection is assigned exactly one corresponding number word, 2) the *stable-order principle* – number names are assigned using a consistent sequence, 3) the *cardinal principle* – the final number assigned when counting is representative of the total quantity of the collection, 4) the *abstraction principle* – any collection of "things" may be counted, and 5) the *order-irrelevance* principle – altering the order in which the objects in a set are counted does not change the outcome of the count. The first three principles govern how to count, while the latter two concern the conditions surrounding counting situations. Gelman and colleagues (Gelman, 1990; Gelman, Meck, & Merkin, 1986; Greeno, Riley, & Gelman, 1984) argued that preschool-aged children displayed knowledge of these principles, even as their ability to use them accurately was still developing. In Gelman and colleagues' view, children's understanding of counting principles paved the way for accurate counting.

These claims that young children possessed implicit understandings of counting principles sparked a great deal of scholarly debate and subsequent research about the source, nature, and development of children's counting. In contrast to Gelman, some researchers argued that understanding of the counting principles *followed*, rather than preceded, children's use of accurate counting procedures (e.g., Briars & Siegler, 1984; Siegler, 1991). Others favored an intermediate perspective – counting skills and understanding developed in an iterative fashion, with conceptual advances supporting procedural skill, and vice versa (Baroody, 1992; Baroody & Ginsburg, 1986; Fuson, 1988). Researchers also questioned whether or not different aspects of counting should actually be considered "principles." For example, the use of a stable, standard number sequence with one-to-one object correspondence might better be characterized as defining accurate counting, whereas understanding that the final count tag provides the numerosity of the set truly constitutes a principled idea that connects the counting process with its cardinal meaning (Baroody, 1992; Fuson, 1988). The nuances of how different researchers conceived of and operationalized investigations of different principles sometimes led to conflicting interpretations of what children understood about counting and about a given task, especially in cases involving deviations from a standard counting procedure.

A challenge in determining what young children understand is that it is often difficult to distinguish between what a child is attempting to do when counting versus what they actually do. For example, fine motor challenges, or difficulty in keeping track of which objects have been counted and which have not may complicate the act of maintaining counting correspondences (Fuson, 1988). Thus, a child may intend to assign a single number word to each object in a collection but skip or double-count some objects. Another child might apply the cardinal principle in combination with a nonstandard number sequence so that accurate use of the principle yields an "incorrect" total. In attempting to address these challenges, many studies (often conducted in laboratory settings with monolingual, middle-class, European-American populations) used of a variety of tasks designed specifically to distinguish between children's understanding of an underlying principle as opposed to their procedural skill to accurately count. In some cases, tasks were designed and administered to simplify the demands of counting for the child (for instance, by arranging objects into a line before asking the child to count). In other cases, children were asked to evaluate the validity of someone else's counting process (a puppet or computer program) who engaged in counting in an unfamiliar or novel way (by beginning with the second object in a row or skipping adjacent objects and returning to them later in the count; Briars & Siegler, 1984; Gelman et al., 1986; LeFevre et al., 2006; Rodríguez, Lago, Enesco, & Guerrero, 2013). While such accommodations and adaptations have been useful in investigating the nuances and conditions of children's use of counting principles, such tasks often isolate discrete aspects of counting from how they are actually used, obscuring children's participation in the overall activity of counting.

Meaningful object counting, rather, is achieved through the use of the three "how to count" principles *in relation to one another*. While some research attempted to outline a developmental sequence in children's relational use of the principles (Fuson, 1992), this topic has received relatively little attention in subsequent research, particularly with respect to counting larger (greater than 20), unorganized collections of objects. Other research documented how as children are still working on the mechanics of counting, they may sometimes violate one aspect of a principle in order to satisfy another. For example, a child may "recycle" previously used number names in an effort to continue counting beyond their known sequence (thereby assigning the same number name to multiple objects), or conversely, a child might cease counting before working through the entire collection because they have run out of unique number names (Baroody & Price, 1983). Whether researchers deem these (conventionally) incorrect but "partially principled counts" (Sophian, 1998) as displays of understanding, or a lack thereof, is often a matter of interpretation.

Further research has unpacked and elaborated upon the principles as originally described by Gelman and Gallistel. For example, accurate one-to-one counting can be interpreted as involving multiple related components, such as coordinating number words with objects, keeping track of which objects have been counted and which have not, and whether or not the tags assigned to objects are truly unique or are repeated later for another object (Alibali & DiRusso, 1999; Baroody & Price, 1983).

In addition to the stable-order of the number sequence, principled ideas also guide how the sequence is extended. The naming scheme for numbers beyond 10 (or perhaps 20[3]) is structured according to the base-ten number system.[4] Rather than memorizing the entire 1–100 number sequence, children learn to use the sequence of decade names (*twenty, thirty, forty*…) in combination with the recurring 1–9 sequence to continue counting. Many children demonstrate that they are attending to this underlying structure even as they are still learning parts of the standard sequence. For example, a child might count from 1 to 29 and continue 40, 41, 42 (skipping the 30s), demonstrating the use of the principled idea within a nonstandard sequence. These "rule-governed errors" provide evidence that learning to extend the counting sequence is not simply an act of memorization, but rather one of sense-making (Ginsburg, 1989).

The relationship between the cardinal principle and broader notions of number and quantity is quite complex and has been the subject of extensive research. Some children provide a "last-word response" consistent with the cardinal principle when asked to count a collection (the *how many* task), but do not generate a set of a given size within their counting range (the *give n* task), leading researchers to conclude that an understanding of how counting relates to number is not wholly captured by children's use of the cardinal principle alone (Bermejo, Morales, & deOsuna, 2004; Fuson, 1988; Mix, Sandhofer, Moore, & Russell, 2012; Sarnecka & Carey, 2008). On the other hand, children's ability to produce a set of objects a given size may represent a more demanding task than counting to find how many (Clements & Sarama, 2015), and thus underestimate children's cardinal number knowledge (Baroody, Lai, & Mix, 2017). Furthermore, a thorough understanding of cardinality would also entail that children understand ideas related to hierarchical inclusion and the successor function – that when counting each new number contains the already counted amounts; thus each consecutive number holds a cardinal value of one more than the previous number (Kamii, 1982; Sarnecka & Carey, 2008). A full exploration of issues related to cardinality (and by extension, research on subitizing) is beyond the scope of this article. Rather, in this study, we focus on the principled ideas within Table 1. These most closely resemble Gelman and Gallistel's three "how to count" principles, though we extend principles related to the number sequence to include recognition of the patterns of the base-ten number system.

While the field has generally moved beyond extreme nativist or skills-based perspectives (Baroody, Lai, & Mix, 2006; Baroody & Purpura, 2017; Clements & Sarama, 2007), there is agreement that learning to count with understanding takes time, and that what may present as incorrect responses to counting tasks often reflect some amount of adherence to counting principles. If the assessment is to document what children know and can do related to counting, extant research suggests that it would be productive to attend to children's use, or partial use, of the principled ideas that underpin meaningful counting. One of the goals of this study is to explore the affordances of extending assessments of counting beyond all-or-nothing accounts of overall performance or principle use to more accurately reflect the development of counting understandings.

Conceptually, the principled ideas listed in Table 1 extend beyond simply learning to count. One can view the counting principles as specific early instantiations of more general number concepts – big ideas that are extended again and again in the learning of mathematics. As they progress through

Table 1. Principled ideas in learning to count.

The Sequence of Number Words
Counting involves using a consistent, ordered sequence of number names (the stable-order principle). Extending the number sequence involves making sense of the patterns of the base-ten number system.
One-to-One Correspondence
Exactly one number from the counting sequence is assigned to each object in the collection (the one-to-one principle).
Cardinality
The last number assigned to an object in counting the collection represents the total quantity of the collection (the cardinal principle).

elementary school, children extend the ideas of place value and base-ten inherent in number words to describe and conceptualize progressively larger (and smaller) numbers. The one-to-one principle is a specific case of the more general notion of one-to-one correspondence, which extends to problem-solving strategies (e.g., direct modeling strategies for comparison problems), and later to the unit and proportional correspondences (multiplication and division, fractions, proportional reasoning, linear functions). Cardinal understanding forms the basis of more sophisticated problem-solving strategies (e.g., counting on) and fundamental ideas of number composition. Viewed in this light, the counting principles are a critical building block for generative understandings of the number. If children are to develop such understanding, challenging, open-ended counting tasks (e.g., larger, unorganized collections) may provide opportunities for children to grapple with the complexities of the counting principles, and in doing so reveal connections to the major mathematical work of the elementary school.

Counting as a social activity

Researchers from a variety of perspectives and traditions acknowledge that children's mathematical understanding is situationally and contextually dependent. Individual children's performance on a given task in a given moment is highly variable (Siegler, 2007; Wager, Graue, & Harrigan, 2015), and children's understanding of counting is best understood in relation to the sociocultural contexts of their development (Saxe et al., 1987). The details of how tasks are posed (e.g., Fuson, 1988; Le Corre, Van de Walle, Brannon, & Carey, 2006), in combination with how children and the interviewer interact together around a particular task are central in determining how a child participates and what counts as competence in a given setting. More than 30 years ago, Steffe and colleagues called attention to the importance of understanding children's interpretations of the tasks in which they were asked to engage:

> Our focus is not the tasks as we conceive of them, but rather the tasks as they are interpreted and solved by the child. The child's "tasks" may be unrelated to the interviewer's "task." Thus, no task by itself can serve as a rigid criterion for a particular operation or ability. We must emphasize that the counting behaviors presented are only samples. They do not in any sense exhaust the behaviors available to any child nor are the children exhaustive of those we have observed. (Steffe, von Glasersfeld, Richards, & Cobb, 1983, p. 46)

In this study, we view children's participation as reflecting their efforts to make sense of the socially constructed nature of counting. Children's participation in a given interview reflects a negotiation of meaning between the child, the interviewer, and a particular task (Parks & Schmeichel, 2014). In investigating young children's counting within the context of an assessment interview, our goal is not to determine whether or not they possess explicit knowledge of a given principle, but rather to explore the details of their counting activity across tasks that vary in their affordances and constraints.

Methods

This study emerged from analyses of a broader assessment that focused on young children's mathematical understanding. We assessed 476 children as part of a larger project that provided professional development in early childhood mathematics designed to support coherence in children's learning opportunities from preschool through third grade. In this article, we report cross-sectional data from fall pretest interviews of preschool students, administered within the first six to eight weeks of the school year, for selected items related to counting.

Participants

We collected data in 50 preschool classrooms[5] across two metropolitan areas, half of which were participating in our professional development program and the other half chosen for comparison. In each classroom, we assessed 10 children (usually five girls and five boys) randomly selected from the classroom roster of those with parental permission. Forty-seven of the 50 classrooms served low-income families as determined by program type or funding source (e.g. Head Start, State Preschool, school district free/reduced lunch percentage of 75% or higher). The programs or school districts across the entire sample were primarily 75–98% English Language Learners, indicating that the majority of the sample was comprised of emergent bilingual children (many classrooms provided instruction in both English and Spanish). The ages of the children assessed ranged from 3 to 5 years (37–62 months), with a mean age of 53 months (see Appendix A).

Measures

For the purposes of this article we focus on three specific counting tasks: 1) counting out loud (as high as you can), 2) counting a prearranged set of eight bears, and 3) counting a collection of 31 unorganized pennies. These three tasks allowed us to attend to the range of children's use of counting principles while varying the complexity of the task. For the items that involved counting objects (bears and pennies), we recorded the child's final response as well as the extent to which the child displayed: a) use of the standard counting sequence, b) one-to-one correspondence between objects and the assigned number names, and c) a response to "how many" that reflected the last number used when counting. Here we highlight the most essential details for understanding how the design and administration of each task provided opportunities for children to demonstrate their understandings of counting, and how their participation in each task was captured and analyzed (additional details are provided in Appendix B). The details of the task design, administration, and coding are critical for making sense of the findings and relating our findings to others in the field.

Count out loud

We chose to begin the assessment with a task commonly administered in school settings and likely familiar to children. The interviewer invited the child to count as high as they could, starting at one. If the child stopped counting before making an error in the standard sequencing, the interviewer prompted once to see if the child could keep going with their counting.

Count eight bears

Assessments of early childhood mathematics often include asking children to count sets within 10 (Ginsburg & Baroody, 2003; Purpura & Lonigan, 2015; Weiland et al., 2012). The interviewer placed eight plastic bears (of the same color) in a horizontal line in front of the child, and asked the child how many toy bears were in the collection (see Figure 1).[6] The interviewer directly assessed the cardinal principle by asking "How many bears?" when the child had finished counting.

Count 31 pennies

We designed this task to engage children in the complexities of counting. The collection was intended to be larger and more challenging than is usually administered to young children in an effort to maximize opportunities to capture their varied mathematical understandings. We specifically chose 31 pennies to provide opportunities for the child to extend their count

Figure 1. The bears task as presented to a child.

Figure 2. The pennies task as presented to a child.

sequence past the twenties and to complicate the application of the one-to-one and cardinal principles. Thirty-one pennies were poured from a bag onto the table in front of the child into an overlapping pile (see Figure 2). The interviewer then asked the child if they could help count all of the pennies. Note that this presentation required the child to do some organizing or moving of the objects in order to successfully complete the task. Upon completion of their counting, the interviewer asked the child how many pennies were in their collection. If a child recounted they were offered a second opportunity to provide a cardinal response (in an effort to mitigate the possibility that the child interpreted the cardinal prompt to imply that they had counted incorrectly).

Procedures

The assessment occurred in the fall of 2014 during the first six to eight weeks of the school year. Children were interviewed individually by an assessor with knowledge of children's mathematical thinking and experience in working with young children. Whenever possible assessors would arrive early and visit informally with children as they arrived at school or ate breakfast. Assessments were conducted in the classroom as a means to provide a more comfortable and familiar setting while engaging with an unfamiliar assessor. Assessors interviewed children one-on-one in a quiet area of the classroom in an effort to allow children to focus on the tasks while also remaining in proximity to their classmates and teachers.

Interviewers conducted the assessment in the child's language of choice (English or Spanish for this sample). Bilingual assessors introduced themselves in both English and Spanish to children who, based upon teacher recommendation, might prefer to be interviewed entirely or partially in Spanish.[7] Maximizing opportunities to gauge understanding sometimes included alternating back and forth across languages, particularly when a child demonstrated confusion or hesitation with a new task. Approximately 43% of the children chose to be assessed either partially or mostly in Spanish. Administration of the entire assessment lasted about 15–20 minutes.

All assessors participated in training sessions to ensure consistent and accurate coding (detailed in the next section). Training sessions included an in-depth study of counting principles, including the reading and discussing of text, and viewing video examples of children's varying approaches and understandings of counting (Carpenter, Franke, Johnson, Turrou, & Wager, 2017). Assessors worked through discrepancies in coding exemplars to arrive at a consensus. By the end of the training sessions, assessors were required to demonstrate accurate coding of a common set of videotaped assessments. In total, 19 assessors (over half of which were fluent or conversant in Spanish) were

Table 2. Coding for counting principles for each task.

Task	Number Sequence	One-to-One Principle	Cardinal Principle	Final Response
Count out loud	Highest number reached using the standard sequence	N/A	N/A	N/A
Count eight bears	Use of standard sequence 1-8 Nonstandard sequence	Accurate use of 1:1 within each of the following intervals: 1^{st}-4^{th}, 5^{th}-8^{th} objects	Used cardinal principle (CP) Did not use CP	Correct (8) Incorrect
Count 31 pennies	Highest number reached using the standard sequence	Accurate use of 1:1 for each of the following intervals: 1^{st}-5^{th}, 6^{th}-10^{th}, 11^{th}-20^{th}, 21^{st}-31^{st} objects	Used CP Did not use CP	Correct (31) Incorrect

trained and conducted interviews. Assessors included the authors, graduate and undergraduate students, and former classroom teachers. Assessors coded while interviewing with the help of a structured coding sheet that listed all of the coding possibilities and a space for additional notes. Assessors checked and completed their coding immediately following each interview with the support of a lead assessor.

Coding and analysis

For each task we considered ways to capture the nuance of children's engagement in counting; we coded for the child's use of 1) the number sequence, 2) the one-to-one principle, 3) the cardinal principle. We also coded whether the child gave a (conventionally) correct final response. Table 2 provides an overview of the coding scheme for each of the three focal tasks.

Number sequence

For the oral count and pennies tasks, the interviewer recorded the highest value counted using the standard sequence and the highest count with a minor mistake (skipping a single number in the sequence or making a single reversal). For instance, if a child counted 1–12, skipped 13, and continued 14–20, we noted that the child was able to produce the number sequence accurately to 12, and to 20 with one minor mistake. As much as possible, the interviewer also attempted to track nonstandard sequences used by children. For the bears task, which limited the counting sequence to only 1–8, we simply marked if the child used a standard or nonstandard sequence. For the pennies task, summary codes were created for different ranges of numbers reached using a standard sequence: within 10, 11–20, and more than 20.

One-to-one principle

We intentionally parsed the collection into intervals that would make it possible to capture emergent use of the principle and marked whether the child applied one-to-one accurately for that interval. For the bears task, we coded intervals for the first (objects 1–4) and second halves (objects 5–8) of the collection. For the pennies task, we coded intervals of approximately 10 consecutive objects (1^{st}-10^{th}, 11^{th}-20^{th}, and 21^{st}-31^{st} pennies), but further parsed the initial interval into the 1^{st}-5^{th} and 6^{th}-10^{th} items. We created this initial smaller interval in order to capture the child's use of one-to-one for a smaller quantity of objects comparable to other large-scale measures of preschool counting (such as TEMA-3, Ginsburg & Baroody, 2003). Accurate use of one-to-one correspondence within each interval was coded independently; it was possible for a child to be coded for not displaying one-to-one for the first interval but to do so for a later interval. We then created summary codes for each task according to the number of intervals coded. For the pennies task, if the child displayed accurate one-to-one correspondence across the entire task (all four intervals), they received the summary code *full*. If the child displayed the use of one-to-one correspondence for one, two, or three intervals they received the summary code *some*. If a child did not show accurate one-to-one for any single interval, they received the summary code *none*. For the bears task, the summary codes *full, some*, or *none* corresponded with two, one, or no intervals coded for accurate one-to-one correspondences.

Cardinal principle

We coded for application of the cardinal principle when the child responded to the "how many" prompt with the last number word they had used when counting, whether or not this was the outcome of fully accurate counting. For example, if a child counted the eight bears 1, 2, 4, 5, 6, 7, 8, 9, and then responded 9 when asked how many bears, they would be coded for use of the principle. We recognize that there are many nuances to children's use of the cardinal principle and that researchers have operationalized the principle in varied ways; our coding focused on the connection between the final number stated when counting and the child's response to how many objects were in the collection.

To illustrate our coding scheme we provide an example drawn from the piloting of the assessment. The transcript in Table 3 shows how 3-year-old Hazel counted a collection of 30 pennies. In making her way through the collection, she moved each penny across the table into a new pile as she counted, occasionally counting a penny twice. After counting all the pennies, she responded that there were 12 pennies in the collection. Using the previously described scheme, she would be coded for reaching 12 using the standard sequence (range of 11–20), accurately applying the one-to-one principle for two of four intervals (the 6th-10th and 20th-30th objects; summary code *some*) and responding to the "how many" prompt in a way that was consistent with the final number used when counting.

We analyzed the data to explore the range and variation in children's use of counting principles. Because the goal of the study was to better understand the details of children's emergent understandings, we made a conceptual decision not to impute missing data related to counting principles. Thus, if any aspect of coding related to the counting principles was missing, that child's data was omitted from the analysis. This resulted in a final data set of 476 children (94.6% of all students assessed). We examined percentages of children who responded in each of the coding categories and how a child's response on one task compared to their responses on the other tasks. We were also interested in children's relational use of the principles within the pennies task, and whether the use of certain principles preceded others (IBM SPSS Statistics, Version 24).

Results

We present the results in two sections. In the first section, we detail children's counting for the collection of 31 pennies and show that assessing the accuracy of children's counting does not adequately represent the depth and complexities of their emerging understandings. In the second section, we compare children's performance on the pennies task with their performance on the oral count and bears tasks and show that children often demonstrate understandings on a challenging task not captured by simpler, more scaffolded tasks. Our overall goal is to highlight how and why attending the nuances of children's counting – particularly the complexities of their emergent use of counting principles – reveals important mathematical understandings relevant for researchers and practitioners.

Part one – children's counting on the pennies task

As previously described, each child was asked to count a collection of 31 pennies poured into an unorganized pile in front of them. One way to assess children's performance on this task would be to

Table 3. Hazel counts 30 pennies.

Interviewer:	Can you help me count these pennies?
Hazel:	Yeah
Interviewer:	How many are there?
Hazel:	1, 2, 3, 4, 5[a], 6[a], 7, 8, 9, 10, 11, 12, 14, 16[a], 17, 18, 19, 11, 12, 14, 16, 17, 18, 19, 11, 12, 14, 16, 17, 18, 19, 11, 12. 12!
Interviewer:	How many are there?
Hazel:	12.

Note: The pennies task was originally piloted with 30 rather than 31 pennies. Video of this particular example can be viewed at http://hein.pub/YCM3.21

[a] denotes a double-count

Table 4. Correct and incorrect responses for the pennies task.

Response	*n*	%
Correct	12	2.5
Incorrect	464	97.5
Total	476	100

evaluate whether or not each child correctly answered that there were 31 pennies in the collection after counting. Table 4 shows children's performance assessed in these terms.

Viewed in terms of correct versus incorrect, children's performance on the pennies task is unsurprising. Only a very small number of children answered that there were 31 pennies after counting their collection. We were interested, however, in capturing the details of what children did and said as they attempted to count such a large set, and how their participation in the task might reflect an understanding of fundamental counting principles. In the following sections, we detail children's performances in terms of their use of the number sequence, the one-to-one principle, and the cardinal principle.

The sequence of number names

We begin first with children's use of the standard sequence of number names. Table 5 displays ranges of numbers reached when counting without making an error. Despite the obvious challenge of counting such a large collection, more than 93% demonstrated some knowledge of the standard number sequence. Within the task, almost 60% of the children extended the count sequence beyond 10, and more than one in seven were able to count accurately into the twenties. While these data were collected near the beginning of the school year, many students were already beyond typical preschool expectations for using the standard number sequence up to 10 (~48 months) or 20 (~60 months).[8]

The most common highest numbers reached using an accurate counting sequence were 14 (*n*= 56; 11.8%) and 12 (*n*= 38; 8.0%). This makes sense, as the subsequent number names, 13 and 15, do not follow the naming scheme of the other teen numbers (as their prefixes are derived from the ordinal rather than cardinal number roots – *thir*teen and *fif*teen rather than *three*-teen and *five*-teen). The frequent omission of 15 is consistent with Fuson's (1988) findings, and the irregular structure of the teens is commonly acknowledged by researchers as a distinguishing feature of learning the number list in English (NRC, 2001, 2009).

The number of sequences used by individual children often revealed the emergent understanding of the patterns of the number system. Table 6 provides several examples of individual children's highest number reached in the number sequence without error, along with the specific number sequence they used. For example, Alyssa counted pennies using the standard sequence from 1 to 14, then continued 16, 17, 18, 19. After reaching 19, she extended the count by repeating numbers, 16, 17, 18, 19, 18. While Alyssa makes some errors in her counting, her errors suggest that she knows much more about the number sequence than simply how to count to 14. For one, while she skipped saying 15, Alyssa otherwise used the standard teen number sequence to 19. Next, in repeating numbers to extend the count, she does

Table 5. Children's use of the number sequence.

Range of highest number reached	*n*	%
21–31 + [a]	74	15.5
11–20	206	43.3
1–10	165	34.7
0[b]	31	6.5

N = 476. Child could count in either Spanish or English. Data are for the highest number reached using standard sequence without error.

[a] It was possible for a child to count beyond 31 if some pennies were counted more than once.

[b] 0 denotes that either the child did not engage in the task, or that their counting began with a number other than 1.

Table 6. Examples of children's use of the number sequence.

Child	Highest number reached without error	Number sequence used when counting pennies
Juan	29	1–29, "twenty-ten," "twenty-eleven"
Celia	28	1–28, 30, 40, 50.
Alyssa	14	1–14, 16–19, 16, 17, 18, 19, 16, 17, 18, 19, 18
Anthony	6	1–6, 11, 12, 13, 14, 16
Daniel	0	8, 9, 2, 8, 10, 11

so (mostly) in sequence, rather than stating numbers randomly. Furthermore, in cycling back to repeat part of the sequence (as we do in repeating the 1–9 sequence within each new decade), she does not return to one. Similarly, Juan's use of "twenty-ten" and "twenty-eleven" to extend his count reveals an awareness of the naming scheme of appending the decade name with the previously used sequence, while Celia's counting beyond the 20s demonstrates knowledge of the sequence of decade names. While Anthony has not yet learned to count to 10 using the standard sequence, he uses unique number names in an ascending order and knows some of the sequences that follows 10. Even Daniel, who did not begin his count with 1, does know some number words and uses these words when counting (rather than, for example, letter names). Taken together, these examples illustrate the ways in which what may present as errors often provide evidence of a child's understanding, and that across each performance level children possess some knowledge of the counting sequence. Simply assessing whether or not children could count to 31 does not capture the depth and nuance of their understandings of the principle ideas of the number system.

One-to-one correspondence

Within the task children also varied in the consistency with which they assigned each object a corresponding number name. Recall that complete use of one-to-one for at least one interval would be coded *some* one-to-one, with a coding of *full* one-to-one for all 31 objects. Table 7 shows that more than 73% of the children demonstrated consistent use of the one-to-one principle for at least one stretch of objects, with 16% of the children maintaining accurate use of the principle throughout the entire task. Similar to the count sequence, many children had already met or exceeded state expectations for preschoolers to demonstrate one-to-one correspondence when counting 5 (~48 months) or 10 (~60 months) objects (California Department of Education, 2008).[9]

Some children demonstrated the use of the one-to-one principle without employing the standard number sequence. For example, Dolores (see Table 8) omitted several teen numbers while counting, but assigned a unique number name to each and every object. Other children accurately applied the one-to-one principle across a subset of the objects but not across the entire set. Ramón assigned a unique number name to each of the first 20 objects (omitting the number 15 in the sequence), but beyond 20 was unable to keep track of which objects had been counted and which had not. It was not always the case that students who demonstrated accurate use of the one-to-one principle for some of the task did so at the onset. For example, Kaylee's coordination of number names and objects was inconsistent initially, but as she continued her use of the one-to-one principle became accurate, likely due to the rhythm of sliding pennies across the table while saying each number name. Even children who did not coordinate objects with number names for an entire interval of pennies still at times provided some evidence of the principle. Jesse, for instance, began sliding pennies one at a time while saying the numbers 1–4, but did not continue to coordinate objects and number names beyond the 4th object. While the coding scheme places Jesse's performance into the "none" category

Table 7. Children's use of the one-to-one principle.

Consistent application over intervals of objects	*n*	%
Full	76	16
Some	272	57.1
None	128	26.9

N = 476

Table 8. Examples of children's use of the one-to-one principle.

Child	Use of one-to-one principle for consecutive objects	Description
Dolores	First through 31^{st} objects	Counted out loud (1–13, 16, 17, 18, 20–34) while sliding each penny across the table from its original pile into a line. Assigned exactly one number name to each penny; used exactly 31 number names.
Ramón	First through 20^{th} objects	Counted out loud while touching unique pennies. Was inconsistent in applying one-to-one principle beyond 20^{th} penny.
Kaylee	6^{th} through 31^{st} objects	Counted out loud while sliding each penny across the table from its original pile into a new pile of "counted" pennies. Was consistent in applying one-to-one principle after early correspondence errors.
Jesse	1^{st} through 4^{th} objects	Counted in coordination with sliding objects across table for first 4 objects, then became inconsistent in matching number words with act of touching and moving pennies.

in Table 7, it is inaccurate to say that Jesse displayed no knowledge of the one-to-one principle. Collectively, children's varied use of the one-to-one principle within the pennies task supports the conclusion that this correspondence is not an all or nothing endeavor, and that a great number of children demonstrate partial understandings of this principled idea.

Cardinality

Children provided a range of responses when asked "how many?" after counting the pennies. Some children responded by restating the last number word they had used when counting while others recounted repeatedly, did not respond, or provided a number other than the last counting number used. Some children provided descriptive responses, such as "a lot." The results in Table 9 show that over 54% of the children correctly applied the cardinal principle by giving a response that matched the number assigned to the last object counted. Note that the majority of correct applications of the cardinal principle did not assign a quantity of 31 to the collection.[10] While the set size of the pennies task exceeded even the largest sets investigated by Fuson, our results support the conclusion that accurate counting is not a requirement for use of the cardinal principle (1988, Chapter 7).

For example, Brian (see Table 10) counted the collection of pennies using an accurate number sequence, but double-counted one penny near the end of his count. When asked how many pennies were in the collection, Brian responded "32," consistent with the final number used in his count. Similarly, Emily counted until reaching the number 10 in her sequence. When asked "how many?" she responded "10." In contrast, Ana's response of "muchas" does not assign a specific quantity to the collection. And yet, one might interpret that her descriptive response does suggest some understanding of the outcome of counting – that the process of counting provides information about the

Table 9. Children's use of the cardinal principle.

Response to "how many pennies?"	*n*	%
Applied cardinal principle	12	2.5
to a count ending with 31	246	51.7
to a count ending with a number other than 31		
Did not apply cardinal principle	218	45.8

N = 476

Table 10. Examples of children's use of the cardinal principle.

Child	Used cardinal principle?	Description
Brian	Yes	Counted 1–32 using an accurate number sequence, but double-counted one penny. Responded "32" when asked how many pennies were in the collection.
Emily	Yes	Counted pennies (skipping 7 in the sequence) until reaching the number 10. When asked how many pennies responded, "10."
Ana	No	Counted into the teens, ending at 16. When asked how many pennies responded, "muchas."
Osvaldo	No	Counted pennies, finishing with the number 20. When asked how many pennies, recounted each time.

amount of objects in the collection. On the whole, children's application of the cardinal principle on the pennies task revealed many shades of preschoolers' emergent understanding of the relationships between counting and cardinality.[11]

The relational development of counting principles

For the purposes of highlighting the details of children's participation in the pennies task, the data presented have necessarily isolated each of the principled ideas that underpin learning to count. Of course, as they engaged in counting pennies, children worked to coordinate the use of each principle with the others. Examining children's relational use of the counting principles provides a window into their concurrent development, and how the use of a given principle did not emerge in the same ways for each child or in a specific sequence across the data set.

For example, Amaya counted "1–13, 15, 16, 18, 20, 21, 22, 23, 24, 25, 29, 20, 21, 22, 23, 24, 25, 28, 29," and stated that there were 29 pennies in the collection. In doing so, she assigned exactly one number word to each of the 31 pennies and gave a cardinal response consistent with the last number assigned. In other words, Amaya accurately used the one-to-one and cardinal principles as she attempted to extend the number sequence. Other children displayed different relational understandings, and the use of one principled idea did not necessarily require knowledge of the others. Jasmine counted using the conventional sequence from 1 to 29, skipping only the number 20. She applied the one-to-one principle consistently for the first 20 objects before making a slight correspondence error. When asked how many pennies were in her collection, she recounted the collection again rather than providing a cardinal response. For Jasmine, the cardinal and (to some extent) one-to-one principles appeared to trail the development of the standard number sequence, whereas for Amaya, use of the cardinal and one-to-one principles seemed to precede use of the standard number sequence. Still, other children displayed early use of one principle but not others. Daniel (shown previously in Table 6) used a non-standard sequence, "8, 9, 2, 8, 10, 11," but touched exactly one penny for each number word. Thus, it was possible for a child to begin to apply the one-to-one principle without knowing even a partial number sequence.

While we did not design this study to map a developmental progression of children's use of the counting principles, our overall data show that children displayed varied relative understandings of the principles on the pennies task. Recall the number of categories reported for each principle: four categories for use of the number sequence (Table 5), three for use of the one-to-one principle (Table 7), and two for use of the cardinal principle (Table 9). Thus, there are a total of 24 possible combinations of principle use. Across the sample, children presented 22 of 24 possible combinations of principle use. For example, some children were still working on the number sequence within 10 in combination with the accurate use of the one-to-one and cardinal principles. Other children displayed knowledge of the number sequence into the 20s in combination with a limited understanding of the one-to-one and cardinal principles. Still, others demonstrated the use of the cardinal principle in combination with a limited number sequence and no use of one-to-one.

Taken together, our data for the pennies task suggest that while children develop understandings of the counting principles concurrently, there is not a specific, "typical" developmental sequence. Rather, children bring different constellations of understandings to the task of counting and can engage productively in counting as they are still learning to use and coordinate different principled ideas. Furthermore, the variety and range of understanding children displayed of each principle highlight the ways in which counting is not an all-or-nothing endeavor, and that such accounts of children's counting are likely to overlook important emergent mathematical capabilities. Attending to the details of children's participation in complex, challenging tasks (such as the pennies task) can provide opportunities to surface children's partial understandings of consequential mathematics.

Part two – children's counting across tasks

In this section, we examine differences in children's performance across different measures of counting. Specifically, we compare children's performance on the pennies task with their

Table 11. Children's use of the number sequence across tasks.

Highest number reached in number sequence	n	%
Counted higher on oral count	169	35.5
Counted higher on pennies task	193	40.5
Reached same number on both tasks	114	23.9

N = 476. Child could count in either Spanish or English. Data are for the highest number reached using standard sequence without error

performance on the other two tasks: 1) counting out loud and 2) counting a small, pre-arranged collection of eight bears.

Recall that for the oral counting task, children were asked to count out loud starting at 1, and when they stopped, were prompted once to see if they could keep going. In comparison to the pennies task, the oral counting task does not require the child to attend to objects as they count, but rather attempts to isolate their use of the number sequence. How did children's use of the number sequence compare across the oral counting and pennies tasks? Table 11 displays children's performance in terms of which task resulted in a higher count.

There was a broad range of performances across the two tasks, with no single outcome accounting for even half of the sample. Slightly more children counted higher when counting the pennies than when asked to count out loud without objects (40.5% vs. 35.5%, respectively), with less than a quarter of children giving corresponding performances across the two tasks. A large sample approximation to the sign test (Conover, 1999) did not reveal a significant difference in the number of children that counted higher with pennies than orally ($Z = 1.21$, $p > .05$) so we cannot rule out that more children counting higher on the pennies task is due to chance or that the pennies task is a "better" indicator of children's knowledge of the number sequence. However, given the ubiquity of the oral counting task (in both researcher and practitioner assessments), children's performance on the pennies task suggests that oral counting may underestimate some children's knowledge of the number sequence. Considering 64.4% of preschoolers counted either the same or higher on what is typically seen as a more challenging task, it is productive to investigate the range and variability in children's counting across tasks. Drawing on the following examples, we provide several possible interpretations of children's varied performances.

An (often unstated) assumption is that isolating the production of the number sequence is an easier task, allowing children to focus only on this one aspect of counting and thus, enable them to count farther than when counting objects. Certainly, this was true in some cases. Efrain, for example (see Table 12), counted up to 14 before skipping and reusing some of the teen numbers on the oral count task. When invited to count pennies, however, he began counting and touching pennies ("1, 2, 3, 4, 5") but then motioned to the rest of the collection and said "a lot." In this case, it may be that in attending to the overall magnitude of the collection, Efrain *chose* to stop counting earlier than on the oral count; rather than producing a count he thought would be inaccurate, he instead responded with a more general quantifier. In other cases, children continued counting but made errors in their sequence as they worked to coordinate their sequence with the objects.

Alternatively, one might interpret that presenting a large collection of objects makes counting meaningful, and provides different opportunities for children to demonstrate their knowledge of the number sequence. Consider, for example, Selena. When asked to count out loud Selena counted using the standard sequence, stopping at 20 even when prompted to try and keep going. However, when engaging in the pennies task she extended her count, counting 1–29, then 20 and finally 21 in

Table 12. Examples of children's number sequences across tasks.

Child	Number sequence used for oral count	Number sequence used for pennies task
Efrain	1–14, 17, 18, 19, 16, 17, 18, 20	1–5
Selena	1–20	1–29, 20, 21
Juliana	1, 2, 3, 4, 5, 7, 8, 9, 10	1, 2, 3, 4, 5, 7, 8, 9, 10

correspondence with sliding pennies across the table until each one of the pennies had been counted. In this case, one might interpret that the presence of the additional pennies provided an incentive for Selena to continue counting beyond where she might usually stop.

Finally, if children "know" the number sequence up to a certain point, it might be expected that they would provide analogous performances across the two tasks. Juliana, for one, used the same sequence across both tasks, counting 1–10 but skipping the number 7 in the sequence both times. Cases such as these, however, were relatively uncommon; 76% of the children reached a different "highest number," depending on the task. The differences between tasks were relatively large for some children. For example, of children who counted higher on the pennies task, 47% had a difference of 5 or more, including 11% whose difference was greater than 10. In sum, children's use of the number sequence was situated in the task. Neither the oral counting or pennies task alone is sufficient to capture children's understanding of the number sequence; assessing children's understanding with only one of these tasks would present a limited view of what children know and can do.

Children's use of the number sequence was not the only aspect of counting that varied across tasks. Recall that children were also invited to count a smaller collection of eight bear counters. In contrast to the pennies task, in which the pennies were presented to the child in an unorganized pile, in the bears task the counters were prearranged – the interviewer stood the bears up in a line, spaced approximately one inch apart, before asking the child to count. Tables 13 and 14 display children's performance across the two tasks in terms of their use of the one-to-one and cardinal principles.

Examining the data across the bears and pennies tasks reveals that children displayed a great deal of understanding related to the one-to-one principle. Cumulatively, more children demonstrated fully accurate use of the principle across both tasks (n= 63) than showed no use of the principle on either one (n= 41). The most frequently paired performances were children who accurately applied the one-to-one principle for the entire collection of eight bears but for only a portion of the pennies task (n= 195). This result would be expected by previous research; larger, unorganized sets are likely to complicate the task of maintaining counting correspondences (Clements & Sarama, 2007; NRC, 2009). On the other hand, this particular combination accounted for less than half of the sample. There were many children who demonstrated knowledge when counting pennies that was not captured by their performance on the bears task. For example, of 70 children who did not demonstrate the use of the one-to-one principle on the bears task, 41% (n= 29) did demonstrate at least some use of the principle when counting

Table 13. Children's use of the one-to-one principle across tasks.

		Pennies Task			
		Full	Some	None	Total
Bears Task	Full	63	195	59	317
	Some	10	51	28	89
	None	3	26	41	70
	Total	76	272	128	476

N = 476. Number in each cell represents the number of children using that particular pairing of one-to-one principle use across tasks.

Table 14. Children's use of the cardinal principle across tasks.

		Pennies Task		
	Applied Cardinal principle?	Yes	No	Total
Bears Task	Yes	234	83	317
	No	24	135	159
	Total	258	218	476

N = 476. Number in each cell represents the number of children using that particular pairing of cardinal principle use across tasks.

pennies – including three children who used the one-to-one principle across the entire collection of pennies! Thirteen children who accurately applied the one-to-one principle for the entire pennies task did not do so with the smaller collection of eight bears.

In some cases, the setup, constraints, and set size of the two tasks may have led children to adopt different approaches. For example, a child might struggle to move their finger to touch or point to each of the eight bears that were prearranged into a line, but with the larger, unorganized collection decide to pick up pennies one at a time from the pile or to slide each penny while counting to keep track. Other children skipped or double-counted objects early in the count, but later achieved a rhythm in coordinating objects with number names as they continued to work through the larger collection. Looking at children's use of the one-to-one principle across both tasks gives a richer view of their understandings in a way that neither task captures individually.

Similarly, assessing children's application of the cardinal principle across both tasks revealed varied performances. Twenty-four children who did not apply the cardinal principle on the bears task did do so on the pennies task. In other words, nearly one in six children who did not provide a cardinal response to "how many" after counting eight bears demonstrated knowledge of the principle when engaging in a more challenging task. Furthermore, the data suggests that a sizeable proportion of children were selective in their application of the cardinal principle. Of 317 children who provided a cardinal response to the bears task, 83 (26%) did not do so when counting pennies. For example, Melanie counted the eight bears accurately and responded that there were eight. When counting pennies she recited the numbers as she arranged them into a line, but stopped saying numbers out loud after reaching 10 even as she continued to arrange the pennies. She did not provide a response when asked how many pennies were in the collection (perhaps because she knew that there were more than she was able to count). This finding – that children's use of the cardinal principle was contextually dependent – stands in contrast to previous research which states that children tend to apply the cardinal principle consistently, regardless of set size (Fuson, 1988; National Research Council, 2009). Rather than first learning a "last-word rule" as a precursor to understanding its relationship to cardinality, our data suggests that inconsistencies in use of the cardinal principle can in some cases reflect an emerging understanding of this relationship.

Taken together, the variability of children's performances indicates that individual tasks do not provide an accurate measure of what young children know and can do. In some cases, children displayed understandings of counting within a challenging task that were not captured by simpler tasks. Our findings here support the conclusions of Wager and colleagues who found that assessments were likely to underestimate what young children were capable of doing (Wager et al., 2015). Furthermore, we posit that children's varied performances are not just a reflection of the tenuous nature of number concepts in early childhood, but also a result of children's attempts to position themselves competently within the constraints of the assessment interview and what they understand the task at hand to be (Parks & Schmeichel, 2014). What may present as limited knowledge or incorrect responses can in fact be a reflection of understanding. We will elaborate on these issues in the next section.

Discussion

In this study, we sought to capture young children's emergent understandings of counting. Our study is unique in pairing large-scale cross-sectional data about children's counting with descriptive accounts of individual children's performances. In bringing to bear detailed data on 476 children's use of counting principles within and across tasks, our study contributes to the field's understanding of what making sense of counting looks and sounds like in preschool, and of young children's surprising capabilities. In particular, unpacking the details of children's counting within a challenging task by coding their partial use of principled ideas revealed that children know a great deal about counting that often goes unmeasured. Attending to children's use of counting principles allowed us to capture children's partial understandings within performances that produced incorrect responses. Furthermore, children often demonstrated knowledge within a challenging task that was not captured by other, simpler tasks intended to capture the same knowledge. In short, young children often know and can do more than we expect, but this understanding – which presents as incomplete, partial, inconsistent, or otherwise fragile – is typically overlooked or unaccounted for in research and in practice. It is thus likely that common approaches to capturing young children's capabilities underestimate their mathematical understandings.

Attending to children's partial understandings is critical in documenting what it is that children know and are able to do, and in providing teachers with information that could guide instruction and the selection of tasks. Many current discussions regarding early childhood mathematics focus on the achievement disparities in terms of class, race, or language status (Barnett & Lamy, 2013). In our study, which took place in classrooms that served culturally and linguistically diverse, economically marginalized communities, many children began the school year already able to count beyond typical preschool expectations. Opening space for children to demonstrate their understandings by, for example, attending to the details of their counting processes, and letting children drive the language spoken during assessment, is critical if assessment is to inform developmentally appropriate early math teaching (Frye et al., 2013; NAEYC & NCTM, 2010; Stipek & Johnson, in press). A more nuanced picture of children's capabilities offers possibilities for policy and practice to better build on children's assets, and to shift storylines in early childhood education that often focus on what young children lack, rather than the diversity of knowledge and experiences they bring with them into school (Colegrove & Adair, 2014; Herbel-Eisenmann et al., 2016).

Our study contributes to the field's understanding of children's thinking about counting in several ways. First, we highlight the affordances of attending to the details of children's counting within a complex task. Second, we call attention to the limitations of common tasks used to assess counting by noting the variability of performances across tasks. Finally, our data detailing the nuance of children's use of counting principles suggests greater attention to their relational use and additional aspects of counting that have been underrepresented in the previous research.

The affordances of challenging, open-ended counting tasks

The current study draws attention to the nature of tasks used to gather information about what young children know and can do. We concur with English (2016) and others in highlighting the affordances of challenging tasks to reveal a broad range of young children's understandings. In particular, our data provides evidence that challenging counting tasks – set sizes that extend the edges of children's knowledge of numbers and require the organization of the objects – afford opportunities to capture understanding that would otherwise go unmeasured. In comparison, simpler or more scaffolded tasks (smaller, pre-organized collections or oral counting without objects) in some cases underestimate children's capabilities. Assessment that includes a broad range of tasks, administered and scored in ways that attend to the nuance of children's engagement in counting processes, provides opportunities to capture children's partial knowledge and embrace variability in ways that better represent what preschoolers' counting actually looks and sounds like. To be clear, we do not believe that the tasks and analyses presented in this study alone would be sufficient to provide a comprehensive picture of young children's counting understanding. A more elaborated account of a child's counting would require not only multiple measures, but assessment over multiple contexts (A. L. Bailey, Heritage, & Butler, 2014; Seo & Ginsburg, 2004). Our hope is to stimulate conversation and innovation among researchers and practitioners regarding how tasks and settings can better open up space for children to display their emerging understandings of the complexities of counting.

Counting as problem-solving

Our findings suggest that it may be productive to conceptualize children's engagement in counting as problem-solving activity, and counting larger, unorganized collections of objects as providing opportunities to problematize the mathematics in ways that are productive for learning with understanding (Hiebert et al., 1996). Conceptualizing counting as problem-solving requires a reconsideration of what are commonly characterized as "mistakes" or "errors" in children's counting. Our data reveal that such instances often provide evidence of understanding, even if the child is still grappling with how particular concepts relate to each other within counting. The child who extends the 20s to include "twenty-ten" and "twenty-eleven" understands something about the patterns of the number system. The child who provides a correct cardinal response after counting eight bears but who recounts when asked how many after counting a large collection of pennies may realize that for the large collection they have not quite counted accurately, and thus understands that it does not make sense to provide a similar cardinal response in this case. Rather than interpreting cases like these as evidence of misconceptions or buggy attempts to apply a procedure that has been directly taught (Brown & Burton, 1978), we offer that these responses in and of themselves provide evidence of sense-making and understanding. Interpreting children's counting in terms of what they understand, however fragile or partial it may be, focuses attention on what children know and can do, in ways that are consequential for continued mathematical development and problem-solving. Such a focus may open possibilities for multiple productive pathways to learning, providing an entry point for embracing the diversity of children's knowledge and participation as they negotiate what it means to do mathematics and by extension, to "do school."

Extending research on the counting principles

Our findings raise several issues related to children's use of the counting principles, and how their relational use might be interpreted in terms of development and making meaning of counting. In particular, our study draws attention to the complexities of interpreting children's use (or non-use) of the cardinal principle. Children arrived at consistent or inconsistent use of the cardinal principle across the bears and pennies tasks in a multitude of ways, many of which seemed to reflect attempts to make meaning of the relationship between the process of counting and its cardinal meaning. Some children (recall Melanie, for example) correctly counted and applied the cardinal principle for the bears task but did not provide a cardinal response following an attempted count of the entire collection of pennies, possibly because they knew that the amount of pennies exceeded the highest number in their count list or because they were unsure if they had counted accurately. Other children (e.g., Emily) counted pennies until they reached the largest number they knew and then stopped, providing a cardinal response for how many pennies they had counted (so far). While we cannot claim to know the intentions underlying children's responses, collectively these cases suggest that children were often selective in their responses to "how many," or that they interpreted the prompt to mean different things. Thus, we conclude that interpretations of how children's use of the cardinal principle relates to a cardinal understanding of number are best made in light of their overall counting activity within (and across) tasks. The situated nature of children's use of the cardinal principle, while consistent with the field's general consensus that cardinal understanding does not develop in an all or nothing manner (Baroody & Purpura, 2017), calls into question an overly sequential characterization of its development. Previous research that has characterized the cardinal principle as a "last-word rule" that children generalize quickly and apply blindly *before* understanding its significance (Fuson, 1988; National Research Council, 2009) would seem an insufficient representation of children's thinking and sense-making in the present study. Rather than examining children's application of the cardinal principle in isolation, future research could explore its situated use in relation to, for example, how the child has attempted to keep track of which objects have been counted, whether or not the child attempts to count the entire collection or stops short, if the child has exhausted or recycled their number word list, and so on.

The complexities of the cardinal principle call attention to children's relational use of counting principles alongside other aspects of counting which have received less attention in the literature. How children impose organization on larger sets, whether they attempt to count the entire collection, how they count different kinds of objects (e.g., plastic bears that can be stood up versus pennies that can be stacked or slid), the affordances of heterogeneous collections to encourage grouping by color or size, are rarely examined in research. These aspects of counting, however, are critical for figuring out how to count a larger collection and for solving problems, and as such present fruitful areas for future research. We caution against, however, isolating these as discrete skills. Rather, what is important is how these features function in relation to the overall activity of counting and meaning-making.

Indeed, our study provides some evidence that the counting principles do not develop in an isolated, sequential fashion. Recall the cases of Amaya (for whom use of the standard number sequence followed accurate use of the one-to-one and cardinal principles) and Jasmine (who displayed a well-developed understanding of the number sequence alongside an emergent understanding of the cardinal and one-to-one principles). While both Amaya and Jasmine evidenced important understandings of counting, their use of the principles would appear to have taken different pathways. This was not only true of children who displayed relatively sophisticated understandings of counting; even children who were just beginning to count (e.g., Daniel) displayed early use of different principles. Thus, our data suggests a relational, concurrent view of development that does not take the same sequence for each child. This is an important topic for future research, as it may be more productive to characterize development in terms of a constellation of possible learning paths rather than as a singular, somewhat linear trajectory.

Limitations

In our study, we attempted to create a context that opened up space for children to show us what they understood about counting. One of those ways was by legitimizing their home language as a way to participate in the assessment interview. We were struck by the depth and variety of things they said and did, not all of which were accounted for by our coding and analysis. For example, we did not specifically investigate the relationships between the language in which the child communicated and the ways they named the numbers and quantities (e.g., Ana's response of "muchas;" Table 10). Given that the interplay between number words and concepts is a central feature of children's early number learning (Mix, Sandhofer, & Baroody, 2005), future research should continue to explore the affordances of instruction and assessment that draws on children's multiple linguistic resources (Foster, Anthony, Clements, Sarama, & Williams, 2018; Solano-Flores & Hakuta, 2017).

Relatedly, our coding schemes represented an attempt to capture some of the nuances of children's counting that might reveal partial understandings of principled ideas. Certainly, other ways of coding and capturing understanding would be possible. Future research that attempts to take up (and improve upon) the coding schemes used in this study would do well to consider the ways in which any coding scheme is somewhat arbitrary, and could easily be employed to identify children's deficits – falling victim to what Turiel and colleagues deem the "more or less syndrome," where a specific attribute becomes privileged as more consequential for future outcomes, leading to a tendency to frame individuals entirely in terms of their possession or more or less of the given trait or ability (Turiel, Chung, & Carr, 2016). In this article, we have necessarily focused our gaze on certain aspects of counting, and attempted to emphasize what young children know and can do, but different and broader notions of competence are worthy of researchers' attention.

Implications for teaching and teacher education

Our study provides several important considerations for practitioners and for those who support and oversee early childhood education. First, the ubiquitous use of oral counting to assess preschoolers' and incoming kindergarteners' understanding of the number sequence is likely underestimating the capabilities of many students. That more than 40% of the children counted farther in the sequence when asked to count a large collection of objects strongly suggests that tasks like these be included in both informal and large-scale (e.g. REMA; TEMA-3) assessment. Our data also suggests that teachers should consider not only "how high" children can count using the standard sequence, but also direct attention to children's emergent understanding of the patterns and structure of the number sequence as they attempt to extend their sequence. In many situations, it may be worthwhile to provide opportunities for children to count into the 20s, 30s, 40s, and so on, where the 1–9 sequence is more transparent, rather than focusing on getting the teen numbers correct before moving on to larger numbers.

More generally, our study speaks to the affordances of letting children embrace the complexities of counting. Overly scaffolding or directing children's engagement in counting (e.g., by requiring that they place their objects into a line or requiring that a child know the oral counting sequence before asking them to count objects) artificially limits their learning opportunities and neglects their agency and sense-making capabilities. By extension, using curricular materials that assume a single, rigid developmental trajectory, or require "mastery" of small sets before moving on to larger collections is unnecessary and likely unproductive. A relational, concurrent developmental view of the counting principles argues against isolated, explicit teaching of the principles in favor of broad, varied experiences in counting. Providing opportunities for children to count a variety of collections consisting of a range of set sizes and materials allows for teachers to learn about a child's understanding and sense-making (Carpenter et al., 2017). What is important is not to provide corrective feedback to the child who is still learning to count, but to attend to the details of children's counting, to recognize what they are beginning to understand, and to (at times) respond in ways that support children to build from their own ideas (Jacobs, Lamb, Philipp, & Schappelle, 2011). This may mean, for example, recognizing that a child's way of organizing their collection reflects mathematical understanding relevant for counting and later mathematical work of solving problems. It might also include broadening notions of what counts as mathematics to include space for children's home language and/or invented number names ("five-teen") when counting, and attempting to learn about children's counting activity outside of school (Aguirre et al., 2013; Turner et al., 2012). More important than counting "correctly" is providing opportunities for children to develop an identity as someone for whom counting makes sense, is enjoyable, and is connected to who they are and who they want to become.

As Parks and Wager (2015) discuss, "knowing how to count is very different from knowing how to teach someone else to count," (p.130) and while a great deal is known about the development of children's thinking about counting, relatively little research has addressed how teachers learn to teach mathematics to young children, how they interpret and adapt curricula, and the ways they intentionally support connections between children's informal and formal knowledge of mathematics. For teacher educators, the three "how to count" principles would seem to provide a level of detail that is both useful and accessible to teachers (Ball, Thames, & Phelps, 2008). Further research should investigate the promise of professional development focused on attending, interpreting, and deciding how to respond to children's emergent understandings of counting principles.

Implications for policy

Our data demonstrate the importance of capturing the details of young children's participation and sense-making. Reducing children's performances to terms of correct or incorrect would tell a different and very limited story about what children know about counting. Given the current emphasis on promoting high quality early learning experiences, it is critical that policy is informed by data that accurately reflects what children know and can do. However, large-scale assessments rarely attend to the details of children's participation and are typically administered and scored in ways that overlook children's capabilities – narrowing what counts as knowing and doing mathematics (Franke, McMillan, Johnson, & Turrou, in press). As large-scale assessment outcomes guide decisions regarding curricula, credentialing, access to and funding for early childhood resources, underestimating children's understanding has material consequences for children, their families, and communities. The details of children's use of counting principles are most certainly "in the weeds," but these weeds are consequential for young people.

Conclusion

In this article, we explored the details of preschoolers' participation in counting and found they demonstrated a range of understandings not typically captured in assessments of young children' mathematical thinking. Our findings highlight the affordances of open-ended, challenging counting tasks and of attending to the nuance in children's sense-making processes. If the current moment's increased focus on early childhood is to be successful in building on the assets of historically underserved groups of children targeted by current reforms, teaching, curriculum, and assessment must be more responsive to what children already know and can do.

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Appendix A. Ages of Children

Age range (in months)	Number of children
37–42	24 (5.0%)
43–48	79 (16.6%)
49–54	160 (33.6%)
55–60	172 (36.1%)
61–66 months	41 (8.6%)

The data provided by schools did not allow us to determine whether or not children had also attended preschool the previous school year

Appendix B. Details of Task Administration

Task: Count Out Loud — Materials: None

Instructions for Assessor

- Say: "Can you count for me? How high can you count? Start at one and tell me."
- If child does not begin to count out loud, repeat prompt and say: "Onnnne" *(using lengthened sound in "counting voice" to encourage the child to begin)*
- If child counts and stops, prompt once: "What comes next? Can you go higher?"

Assessor Coding

- Assessor has counting sequence printed on assessment sheet for ease of note taking for starting point, stopping point, skipped numbers, etc.
- After child finishes counting (with prompts), assessor indicates the highest value counted without an error and the highest counted with only a minor mistake of skipping or reversing one number

Example

A child counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Assessor prompted for more. Child continued 11, 12, 14, 15, 16, 18, 19 and stopped there.

- *Highest error-free: 12*
- *Highest minor mistake: 16*

Task: Count eight bears — Materials: 8 counting bears (monochrome)

Instructions for Assessor

- Place 8 bears in horizontal line on table in front of child, spaced equally approximately one inch apart.
- Say: "Please tell me how many toy bears I have. How many bears?"
- Listen and watch as child counts. When child is finished counting, prompt: "How many bears?"
- If child counts the bears again, prompt once more: "How many bears?"

Assessor Coding

- Assessor marks whether the child counted the sequence 1–8 correctly or not
- Assessor indicates in which interval does the child demonstrate one-to-one correspondence (1^{st}-4^{th} items, 5^{th}-8^{th} items, no interval, both intervals)
- Assessor marks whether the child demonstrates the cardinal principle (responding the last number counted to the question "How many?", regardless of it the response is 8 or not; a child who continues to count when prompted "How many?" receives a "no cardinal principle").

Task: Count 31 pennies	Materials: 31 pennies in a plastic baggie

Instructions for Assessor

- Carefully pour out baggie of pennies onto desk in front of child so they are in an unorganized pile with overlapping items.
- Ask: "Can you help me count these pennies?" Listen and watch as child counts. When child is finished counting, prompt: "How many pennies?"
- If child counts the pennies again, prompt once more: "How many pennies?"

Assessor Coding

- Assessor has counting sequence printed on assessment sheet for ease of note taking for starting point, stopping point, skipped numbers, etc.
- Assessor notes the error-free and minor-mistake values (see *Count out loud* task)
- Assessor marks which one-to-one correspondence intervals were completed correctly (1^{st}-5^{th} items, 6^{th}-10^{th} items, 11^{th}-20^{th} items, 21^{st}-31^{st} items)
- Assessor indicates whether the child demonstrates the cardinal principle
- Assessor whether or not the child attempted to organize the collection, and whether this involved moving and/or lining up objects.

Example

A child moves pennies from original unorganized pile into a new pile, saying the counting sequence correctly from 1–19, then repeating teen numbers out of order after 19. The child works quickly and does not consistently say one number for each item for first 9 items, but then slows down and carefully counts and moves each penny from the 10^{th} *penny on. When asked "How many?", the child replies 100, which is the not the last number counted.*

- *Highest error-free: 19*
- *Highest minor mistake: 19*
- *One-to-one correspondence:* 11^{th}-20^{th} *and* 21^{st}-31^{st} *items (coded value: some)*
- *Cardinal principle: no*
- *Organizing: slides counted pennies over into new pile*

Notes

1. We intentionally use the broader term "understandings" (rather than understanding) to call attention to the rich and varied forms knowledge young children bring with them into school settings. A child may understand multiple ideas and practices related to, for instance, what counting is and what it means to count.
2. In this article, we focus primarily on children's counting of discrete, manipulable objects. Object counting is but one form of counting that young children engage in. They can count verbally by reciting the sequence of number words (e.g., "uno, dos, tres, quatro…" or "one, two, three, four…") out loud or in their head. They can count images or objects that cannot be manipulated (dots on a card, trees at a park) or touched (people on the other side of the room, buildings in the distance). They can count quantities that cannot be seen but can be otherwise perceived (hand claps, shoulder taps) or conceived of (family members who are not in the room). They may also count parts of objects as individual units (bagel halves, pieces of a broken pencil) or relate them to the whole.
3. Languages vary with respect to how transparent the spoken names of the numbers map onto the underlying base-ten structure. In Spanish, for example, the value of the teen numbers as 10 and another number becomes explicit only following 15 (*dieciséis, diecisiete*…). In English, the value of the teen numbers is obscured by naming the ones before the 10 (leading some children to write, for instance, *17* as *71*). Furthermore, *thirteen* and *fifteen* (and *thirty* and *fifty*) have ordinal (third, fifth) as opposed to cardinal (three, five) roots. In Mandarin Chinese, by comparison, the names of the numbers and their base-ten value are synonymous; the spoken name for 23, er-shi-san, directly translated to English would be "two-ten-three."
4. We recognize that other cultures and groups of people (such as Oksapmin communities in New Guinea) have developed number systems that draw upon different grouping structures (Saxe & Esmonde, 2005). We focus here on the base-ten number system due to its privileged role in schooling and as it provides access to systems of power.
5. We recognize the politicized nature of terms such as preschool, Pre-K, and child care hold in debates about early childhood education (Wong, 2014). In this article, we use the term preschool in a general sense to describe public programs in which children enroll prior to kindergarten. Our sample includes nine classrooms designated Transitional Kindergarten (TK), a one-year program housed in elementary schools. Children who are four years old and will turn five between September 2 and December 2 of the school year are eligible to attend TK (California Department of Education, 2018).

6. The specific objects used to elicit counting in this study (bears and pennies) shared certain characteristics but varied in others. For example, each set consisted of like objects (as opposed to bears of varied colors/sizes or different kinds of coins), and was likely to be objects with which that children had some prior experience. On the other hand, bears may be placed in different ways on a flat surface ("standing up" or "lying down") whereas pennies are likely to roll or fall over if placed on their side. Relatedly, pennies may be stacked upon other pennies, while bears do not permit this arrangement. It is important to note that when counting an unorganized collection of bears, some children will keep track of which objects have been counted by standing up each bear as they count. This particular approach to the bears task was not available to children in this study as the bears were prearranged by the interviewer. This choice in administration was made in an effort to achieve some consistency with previous research, although we note that unorganized sets potentially offer different kinds of affordances.
7. We recognize complexities, challenges, and possibilities of assessing children's mathematical understanding using their home language. The heterogeneity of emergent bilingual children's cultural, linguistic, and economic histories, range of facility in communicating using their home language, whether they receive instruction in their first language, and issues regarding translation and the specific content assessed all shape the administration and utility of such assessments (National Academies of Sciences, Engineering, and Medicine, 2018; Solano-Flores & Hakuta, 2017).
8. We investigated the relationship between children's age and their enactment of counting. There was a statistically significant correlation between age (in months) and the highest number reached in the number sequence ($r = .314$; $p < .05$). However, as age accounted for less than 10% of the variance on this aspect of the task ($r^2 = .098$), we do not disaggregate data on children's use of the number sequence in terms of age here or for the two principles that follow.
9. As with the number sequence, there was a statistically significant correlation between children's age and their use of the one-to-one principle ($r = .195$; $p < .05$). For this aspect of the task, age accounted for less than 4% of the variance ($r^2 = .038$).
10. Of the 12 children who provided a cardinal response of "31," only eight arrived at this response through both a standard number sequence and full one-to-one correspondence. Thus, a "correct" response in Table 1 did not necessarily mean that children demonstrated completely accurate use and coordination of the counting principles.
11. Similar to the other principles, while age was significantly correlated with children's use of the cardinal principle ($r = .211$; $p < .05$), it accounted for less than 4% of the variance in children's performance ($r^2 = .039$).