

## **Designing Applets that Instantiate Effective Mathematics Pedagogy**

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Despite school districts making increasingly larger investments in interactive teaching technologies, there is scant research documenting the degree to which teachers and teacher candidates use these tools to support meaningful discussions. For example, research has shown that mathematics classes featuring interactive whiteboards often include an increase in superficial questions, but a decrease in cognitively demanding questions. This report describes reflections from two cycles of developmental research that involved creating and refining a computer-based applet for reasoning about the relative magnitude of fractions. The activity stemmed from a cognitively demanding task used in a face-to-face setting and involved placing sets of fractions on a number line. Results from surveys conducted during both pre- and in-service teacher professional development classes served to inform the cyclic process of the applet design. User feedback has indicated that features such as multiple entry points and non-judgmental feedback enhance users' experiences but non-interactive aspects, such as written reflection questions, do not. These results and the accompanying design framework can inform teachers and teacher educators looking for ways to design and evaluate mathematical activities that leverage the features of modern interactive technologies.

## Introduction

Over the past several years, K-12 school districts have invested heavily in sophisticated teaching technologies, such as computer projectors, tablet computers, and interactive whiteboards (Lerman & Zevenbergen, 2007). These investments pose a challenge for teacher educators: How can teacher educators best support pre- and in-service teachers to develop habits of mind for leveraging the various affordances of new technologies *in educationally productive ways*? This report describes results from two cycles of developmental research that are part of a larger, ongoing effort to create mathematical applets to enhance teacher professional development courses<sup>1</sup>. The ongoing research focuses on the degree to which various features of interactive applets support rich mathematical thinking. This report describes the process by which a number-line applet was designed for online and in-class use. Reflections on this process are offered as recommendations for how teacher educators can help pre- and in-service teachers design and choose interactive applets that leverage the communicative features of various classroom-based technologies and which features of online applets enhance students' own explorations.

## THEORETICAL AND DESIGN FRAMEWORKS: A SOCIAL VIEW OF LEARNING

The evolution of computer-based instructional activities can be seen as a quest to improve communication—both between students and the teacher as well as among students themselves. Early efforts placed computer tutorials in the role of “indefatigable drillmasters” (Snyder & Palmer, 1988, p. 75) who forced students to work individually but classroom-based researchers began noticing that communication among students in a computer lab was particularly beneficial to both the students receiving the advice as well as those who were giving it (cf. Dugdale, 2008; Harel & Papert, 1990; Lampert, 1993). For example, Dugdale described the evolution of her *Green Globes* algebra software by stating that she first envisioned students working individually at their own computers to create various functions that would hit as many targets (“globs”) as possible. However, her classroom-based ob-

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1 The authors of this report all served on the design and research team. The first author was the lead programmer and developer of the applets, the second author taught the preservice teachers, and the third and fourth authors were instructors in the professional development program who taught in-service teachers.

servations revealed that students tended to be natural conversants who enjoyed sharing various glob strategies. This realization led her research and development team to think of novel ways to expand students' involvement within their mathematical community so that such communication patterns became normative. One of the most successful innovations was the idea of leveraging a computer network to store and display work in a shared space. When students were able to view other students' successes, the evolution of functions generated by struggling students was impressive. Students appeared to be developing a deeper understanding of functions and how various parameters affected the graphs by engaging in practices that their peers had suggested. When students were able to view other students' successes, their understanding of functions expanded. By engaging in practices suggested by their peers, students more deeply understood the concept of functions and the effect of various parameters on graphical representations.

Dugdale's (2008) shift in design goals aligns with a shift in epistemologies assumed by many educational researchers worldwide. During this time frame, socially situated researchers were advancing the view that classrooms should be perceived as microcultures and therefore the process of learning should be perceived as social in nature (Boaler, 1998; Brown, Collins, & Duguid, 1989; Cobb & Yackel, 1996; Lave & Wenger, 1991). In her recent work, Sfard (2008) fueled this philosophical trajectory by addressing the question of how a social perspective, which focuses on cultural practices within microcultures, provides a lens to study student learning. She addressed this question through a perspective of "commognition," a term she coined to indicate that the processes of communication and cognition "are two facets of the same phenomenon" (p. 9). Although the prevailing epistemologies may disagree on the origins of knowledge and the processes by which knowledge comes to be "shared" (c.f., Lerman, 1996; Steffe & Thompson, 2000), the critical point is that if learning occurs through communication, then the way to improve students' learning is to focus on enhancing the types of conversations in which they engage. For instructional designers and teachers, the implication is that any effort to improve communication should focus on creating provocative didactic objects (Thompson, 2002) that have the potential to support deeper and more conceptual conversations to occur in a classroom.

### **Creating Didactic Objects to Support Engagement**

Thompson's (2002) notion of a didactic object serves as a beacon for in-class instructional design. It suggests that the teacher must create pivot-

al tools that can support novel and robust discussions with multiple entry points and a well-conceived exit point (what Thompson, citing Glasersfeld, referred to as a “conceptual analysis”) (p. 201). For example, Thompson discusses how a conceptual analysis of fractions may involve thinking about unit fractions as composite units. In order to support this view, Thompson designed a simple diagram showing three of five shaded circles. This diagram became a didactic object when he used it to challenge his students to think in novel ways. For example, he asked, “Do you see  $\frac{3}{5}$  of something?” and then, “Do you see  $\frac{5}{3}$  of something?” Thompson stressed that the diagram, in and of itself, is not a didactic object. It becomes one when it is enacted in the social situation of a classroom to support novel conversations in which the students become aware of the critical role that units play in fraction representations.

The previous example highlights Thompson’s (2002) point that didactic objects need not be complicated, but they must perturb, or stimulate, new ways of thinking. One of the ways that technology can support this endeavor is to serve as a “what if” exploratory microworld where students can make hypotheses and then explore unintended outcomes to stimulate resolution. For example, Thompson (1994) described the Over and Back software program he designed, in which students are asked the following types of questions: If a rabbit travels “over” a given distance at, say, 4 meters per second, and then travels back at another speed, say, 8 meters per second, will a turtle, traveling at 6 meters per second tie the rabbit by the end of the race? Students’ natural inclination is to assume that the average of two ratios is the arithmetic mean. Therefore, they are perturbed when the two do not tie. This is an example of how a technology can be used to create surprise and spur resolution through whole-class, small-group, or individual exploration and therefore serves as a great example of how well-designed applets can serve as didactic objects.

### **Mathematical Task Framework: Designs Consistent With a Social View of Learning**

Designing provocative activities such as those in Over and Back (Thompson, 1994) is neither easy nor straightforward. One design framework that has proven helpful in this endeavor is the mathematical task framework created by Stein, Smith, Henningsen, and Silver (2000). The cornerstone of the mathematical task framework is the importance of creating tasks that have *high cognitive demand*. This design theory has been used

for several years to help pre- and in-service teachers develop, analyze, and enact rich problems for whole-class discussions. For example, the authors of this report have been using the framework for many years to lead professional development (PD) workshops for in-service teachers. Teachers in these workshops have defined cognitive demand to be the extent to which a particular task invites the learner to engage in mathematics, which connects ideas and representations and promotes interaction with the problem. A mathematical task that would be considered to have low cognitive demand invites little thought and may be solved algorithmically. In contrast, when learners engage with a problem that poses high cognitive demand, they feel as if they have to sort through their mathematical knowledge to navigate the problem to create connections among various mathematical ideas. Moreover, solving the problem leaves a sense of ownership, satisfaction, and deeper understanding.

It is tempting to conflate cognitive demand with perceived mathematical difficulty. One could argue that the quadratic formula may be difficult to memorize, but restating it is a memorization task, and using it without making mathematical connections to the meaning of roots keeps it at a low level of cognitive demand. On the other hand, finding the area of an irregular-shaped object may not be arithmetically difficult, but it can be cognitively demanding to mentally manipulate and break down the shape into more familiar, simpler shapes for which the area is easier to find. In both these cases, the focus is placed on the intended nature of the solution strategy, not the procedural component of executing it.

It is critical to note that a major assumption of the mathematical task framework is that the cognitive demand of an activity can change when the activity is enacted. This is consistent with a social perspective on learning that suggests that premade activities do not stand alone; their effectiveness and rigor are uniquely determined by how they are realized in a classroom and interpreted by students working within the cultural practices of a particular setting (Cobb & Yackel, 1996). In fact, P. Cobb (personal communication, September 18, 1998) argued that researchers might not speak of activities being “implemented” in a classroom because this suggests all implementations are the same. Instead, he suggested that activities should be researched in terms of how they are “realized” or “enacted” by the participants in different classroom microcultures.

Given this perspective of enacting rather than implementing an activity or didactic object, the mathematical task framework is particularly useful because of its recognition of the many different components that influence mathematics instruction. As the authors of the mathematical task framework stated,

Mathematical tasks and the cognitive demands they place on students comprise but one piece of a much larger pedagogical/mathematical puzzle. We claim only to provide a framework for analyzing practice, not the framework. After learning to use the Mathematical Tasks Framework as a guide for reflection, teachers will, it is hoped, be able to move on to the use of additional frameworks. . . as needed. (Stein et al., 2000, p. 38)

Though developed based on face-to-face interactions in a classroom environment, the mathematical task framework is a useful tool for reflecting on the development and enactment of applets as well. It focuses generally on three components of math instruction. First, there is what could be called Stage 1: the task as it appears in the curriculum. For the professional developers who used the framework prior to development of online applets in the current project, this stage and the accompanying *Task Analysis Guide* ([TAG] described in further detail in *Task Analysis Guide*, below) has been useful for helping teachers to identify high-level tasks in their curriculum. For creating online applets such as the number-line applet, the *Task Analysis Guide* has been useful in promoting conversation about necessary design features for a cognitively demanding interactive applet.

The next two stages of the mathematical task framework take into account the reflexive nature of teacher and student interactions in the classroom. Stages 2 and 3 account for the teacher's and students' influence, respectively, on how a task plays out in the classroom. As a task is enacted, it runs a risk of diminishing in rigor. This might occur, for example, if the teacher gives too many hints or, in the case of applet development, the computer simply gives the answer without any explanations or scaffolds. The students' work on the task, or interaction with an applet, may either challenge them in the ways intended, thus maintaining a high level of cognitive demand, or fail to do so, thus diminishing the cognitive demand of the task.

Whether considering classroom scenarios or applets, each of the stages influences what the students learn and the depth to which they learn it. In other words, the social practices that evolve as the teacher and students work through the tasks can serve to either maintain high cognitive demand or diminish cognitive demand (for example, if the students and teacher end up routinizing the task and playing the school game of demonstrate and copy).

## Task Analysis Guide

In order to guide the process of assessing the initial cognitive demand of various tasks, Stein, et. al. (2000) created the *Task Analysis Guide* (TAG). Within this categorization scheme, low-level tasks are characterized by memorization and the application of procedures without connections to the underlying mathematical ideas. Examples of low-level computer-based tasks might include “drill and kill” or flash-card type exercises that do not involve making novel conceptual connections or deep mathematical thinking. It is important to note that these software programs do have a place in classrooms, but they are not productive for stimulating communication between teachers and students or promoting novel ways of thinking about the underlying mathematics. Moreover, they do help teachers project how the mathematical arguments might play out in a classroom discussion, and hence would not serve as didactic objects either.

Higher demand cognitive tasks involve the application of procedures that do have connections to the underlying mathematics and the actual doing of mathematics. These tasks are designed to focus students’ attention on the use of procedures for developing deeper levels of understanding and require some degree of cognitive effort. Tasks at the highest level require students to apply nonalgorithmic thinking and demand self-monitoring or self-regulation of one’s own cognitive processes.

In keeping with the view that enactment is social in nature and not solely determined by the design of the task itself, Stein et al. (2000) provided a list of factors to help pre- and in-service teachers evaluate the extent to which high cognitive demand was maintained as the tasks are enacted in the classroom. Table 1, which is presented in the following section, illustrates how this list of factors of maintenance and decline aligns with a framework for designing educational applets for online use.

## Adapting the Framework to Design Technologically Based Activities

The authors of this report work as both mathematics task developers and teacher educators under the auspices of a professional development collaborative (PDC). Over the past several years, the teachers in districts that contract for PDC programs have expressed an increased interest in online or hybrid teacher professional development courses. In order to address this request while maintaining the coherence of programs, it became important to develop computer-based interactive applets that could be used during face-

to-face sessions as well as for further online study. The development team sought design principles for online applets that would complement the ideas of the Stein et al.'s (2000) mathematics task framework. The IDEA framework (Underwood et al., 2005), which emerged from reflections on the design of many online applets through the math forum, proved helpful in this regard because it offered applet suggestions that complemented the tenets of the cognitive demand. These parallels are illustrated in Table 1 by comparing a subset of the factors of maintenance and decline with some of the design elements mentioned in the IDEA framework for applets (Underwood et al.). The following sections describe how these principles were applied to the design of the number-line applet.

**Table 1**  
PDC Design Framework (PDC-DF)

Factor	Factors associated with maintenance of high-level cognitive demand (Stein et al., 2000)	Applet design principles (from IDEA framework by Underwood et al., 2005)
1	Tasks build on students' prior knowledge to avoid placing students in situation where they cannot engage.	Design applets that allow students to use various different tools for solving a problem (i.e., multiple entry points).
2	Scaffold student thinking and reasoning to avoid routinizing problematic aspects of entry.	Provide cybernetic scaffolds, such as hints or models.
3	Teacher or capable student models high-level performance.	Use dynamic multiple representations and models.
4	Task involves sustained press for justifications, explanations, and/or meaning through questioning and feedback.	Create opportunities to make predictions, commit to them, and examine outcomes.
6	Students are provided with means for monitoring their own progress.	Give appropriate status feedback.
7	Sufficient time to explore (not too little, not too much), but tasks should be engaging and personally meaningful.	Avoid busy work; computer should do low-level tasks such as filling in squares on a grid or computing sums.

## Applying the Framework to the Design of the Number-Line Applet Series

The following section describes how the PDC-DF guided the creation of a series of applets developed to be didactic objects that could support both in-class discussions and teacher participants' consequent efforts to use multiple, appropriately selected strategies to refine their understandings of rational number relations.

### A Conceptual Analysis of Fractions on a Number Line

Rational number is one of the main foci of the many elementary teacher professional development programs. In face-to-face workshops with teachers, the instructor and participating teachers typically discuss elementary students' difficulties understanding the relative sizes of fractions, decimals, and percents, and ways to combat various student misconceptions. For example, research shows that when children are faced with a situation demanding the comparison of two fractional quantities, they generally do not have any mental imagery to conceptualize the relative sizes of denominators to partitions of those denominators. Most often, they draw area models such as pie diagrams, but this does not help them to compare the relative sizes of several different fractional amounts at the same time. Their other recourse is to rely on a memorized procedure such as cross-multiplying, a binary operation with low cognitive demand that is, at least to many students, devoid of meaning and often misapplied. The teachers in the professional development course discussed other representations for fractional quantities, such as the number line to emphasize linear distance as a comparative measure. In short, the conceptual analysis that guided the number-line activity was to support the emergence of multiple linear strategies for comparing fractional distances, such as comparing them to a common benchmark that is easier to find (e.g.,  $\frac{1}{2}$  or  $\frac{1}{3}$ ) or by comparing the relative magnitude of two different unit fractions. For example, one way to compare  $\frac{9}{12}$  and  $\frac{8}{14}$  is to compare each to  $\frac{1}{2}$ :  $\frac{9}{21}$  is less than  $\frac{1}{2}$  because 9 partitions is less than half of 21 partitions. In contrast, 8 partitions of  $\frac{1}{14}$  is greater than  $\frac{1}{2}$ , and therefore  $\frac{8}{14} > \frac{9}{21}$ .

There were three primary motives for developing an online applet to accompany this number-line activity that has, in the past, been realized in earlier PD sessions via string and note cards containing sets of fractions to or-

der. The first motivation for the applet was that the instructors in the design group who have taught the face-to-face PD courses noted that after engaging teachers in conversations about various strategies, their understandings are initially very fragile and hence the instructors believed that an online applet would provide much needed engagement and practice opportunities. In fact, the instructors noted that it was common for teachers to report feeling comfortable about various strategies during the workshop, but then when completing homework assignments or working in class the next week, they could not remember how to decide which strategy would be most practical for reasoning about the size of the fraction. The PD instructors feared that if the in-service participants could not remember themselves, the likelihood that they would be able to pass on fraction understanding using various reasoning strategies to their students would be small.

Two other motivations for developing the applet were (a) the hope that teachers would use it in their classrooms, and (b) the view that the activity of placing fractions on a number line supports more sophisticated reasoning about other related topics, such as partitioning intervals and representing fraction and decimal equivalents. By supporting the evolution of tools for imagining dynamic ideas, such as the relation between the size of intervals and the number of intervals, teachers gain insights into previously static ideas. From a social point of view, one's thoughts, actions, and experiences are culturally mediated by the tools and activities in which one engages. Although this may seem an obvious conclusion, it suggests that educational tools and the practices that emerge as they are enacted will have profound effects on student learning.

## DESIGNING THE INTERFACE

To begin the applet design process, the PD instructors on the design team were asked to reflect on the types of pedagogical moves they use in classes to maintain high cognitive demand. One stated that he begins by giving the teacher participants sets of three folded index cards containing fractions and asks them to place the quantities in order on a clothesline. The clothesline metaphor is purposely used to suggest that teachers are supposed to focus on the ordinality of the fractions only, and not to consider their relative placement on a number line (Bay, 2001). During this clothesline exercise, the instructor points out the importance of using multiple strategies depending on the fractional values. Once the participants have discussed various strategies for comparing fractions, the instructor shows an unmarked

number line and again elicits strategies for placing the fractions in relatively accurate positions. Prior to the development of the applet, the instructor used an overhead slide projector and cards so that he could reveal each of the fractions individually.

One of the instructor's pedagogical methods for maintaining high cognitive demand was to ask the teachers to orient their guesses by defining benchmarks on the number line, such as 0 and  $\frac{1}{2}$ , and then to consider the relative size of the unit fraction as a measure of how close or far away to move from the benchmark. For example, if the participants are asked to order the fractions  $\frac{4}{9}$ ,  $\frac{8}{15}$ , and  $\frac{14}{29}$  the instructor may ask them to first identify where  $\frac{1}{2}$  is, and then to consider the distance of the fraction from  $\frac{1}{2}$ . The choice of an odd-numbered denominator leads to conversation about how  $\frac{1}{2} = \frac{4.5}{9}$  and the reasons that this second representation can be challenging.

### **Applet Features Designed to Instantiate Pedagogical Moves**

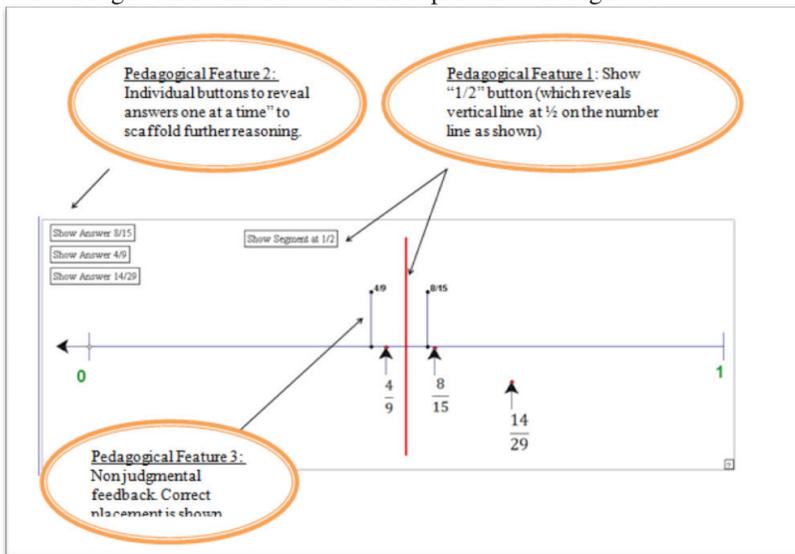
The task of supporting the development of this fragile reasoning via an online applet involved reflecting on the pedagogical moves described above.

*Pedagogical Feature 1: The “show  $\frac{1}{2}$ ” button.* This feature was designed to help users orient themselves to the number line and give them a hint as to one strategy that might be used. As such, it was designed to provide multiple entry points and a way for users to monitor their own progress; two factors associated with the maintenance of high-level cognitive demand.

*Pedagogical Feature 2: The “reveal one at a time” button.* A second interface feature that was designed to mimic the instructor's efforts to maintain cognitive demand as the task was enacted by students was the inclusion of individual “show answer” buttons. The theory behind this design was that the added reflection time would serve to maintain the cognitive demand by scaffolding users' thinking so that they would appeal to strategies related to underlying mathematical ideas (e.g., relative sizes and fractional quantities, such as  $\frac{2.5}{9}$ ) as opposed to relying on memorized procedures (see PDC-DF Factor 2 in Table 1). This feature was also designed to provide further support for self-monitoring through the computer's visual feedback, which aligns with Factor 6 of the PDC-DF.

*Pedagogical Feature 3: Nonjudgmental feedback.* A third feature of the activity was the purposeful use of visual feedback designed to support

self-monitoring. Instead of providing judgmental feedback in the form of “You are correct” or “You are too far away,” the applet allows participants to make their own determination. That is, the actual answer is revealed in response to a click on each “show answer” button, but it is left to the user to determine how close a correct answer needs to be. The underlying design goal was to shift the locus of authority from the computer to the user (Bowers & Nickerson, 2001). This feature is also consistent with the TAG’s (Stein et al., 2000) emphasis on the importance of a sustained press for justifications in that the user has to justify the degree to which his answer is close enough. These three features are represented in Figure 1.



**Figure 1.** Interface of number-line applet as it might be enacted with various pedagogical features.

*Pedagogical Feature 4: Reflection and strategy discussions.* Another cybernetic instantiation of the instructor was the inclusion of thought questions designed to stimulate reflection on the problems and what strategies might be used. For example, one reflection question asks, “Are all of the fractions near any particular benchmarks?” Similarly, the summary statements at the end of each set discuss a possible solution strategy as the teacher might summarize after a given class discussion. This feature aligns with Factors 3, 4, and 5 of the PDC-DF (see Table 1).

*Pedagogical Feature 5: The use of strategic sets of fractions.* A fifth feature of the overall sequence that was designed to maintain a high level of

cognitive demand as the activity is enacted in a classroom is the specific sequencing of fractional values. Presented in Table 2 is the process by which each successive set of fractions lends itself to a slightly more sophisticated strategy.

**Table 2**  
Sequence of Fraction Sets

Set	Step	Step	Step	Strategy
1	Show $\frac{5}{7}$	Show $\frac{3}{7}$	Show $\frac{5}{7}$	Same denominator strategy
2	Show $\frac{4}{8}$	Show $\frac{4}{7}$	Show $\frac{4}{10}$	Same numerator strategy
3	Show $\frac{4}{9}$	Show $\frac{8}{15}$	Show $\frac{1}{2}$	Close to benchmark ( $\frac{1}{2}$ )
4	Show $\frac{9}{10}$	Show $\frac{6}{7}$	Show $\frac{15}{16}$	"Missing piece" strategy
5	Show $\frac{1}{x}$	Show $\frac{2}{x}$	Show $\frac{3}{x}$	Same denominator strategy, with link to algebra

Based on the TAG (Stein et al., 2000), these features suggest that the number-line activity can be classified as a highly cognitively demanding task as it is envisioned. The overall research question is to determine whether or not the activity maintained this high demand when it was enacted on-line. In particular, the design team investigated the following concerns:

1. Did the activities seem challenging to the users?
2. Did the cybernetic features that were designed to model the instructor's moves help maintain high cognitive demand by encouraging the users to try different (more expedient) strategies?
3. Would the participants use this applet in their own classes (thereby supporting their development of habits of mind for using newer technologies)?

## METHODS

The method used to develop and assess various iterations of the professional development materials falls within the broad practice of *design-based research* (cf. Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schau-

ble, 2003). The process begins by conducting a thought experiment during which the design team reflects on the current version of the course materials and envisions areas in which dynamic applets could be supportive. These thought experiments were described in the preceding sections of this report. Beta versions of these applets are then designed and presented to the larger research project team for critique. After revisions are made, the applets are piloted and their use as didactic objects is analyzed by studying the practices that evolved during that particular enactment.

This report focuses on the result of two enactments (Study 1 and Study 2) within the larger cycle of developmental research, as shown in Figure 2.

	Audience/setting	Source	Reflections	Changes in activity/ applet or instance
Study 1	Preservice teacher methods class (Online session with later in-class follow-up) Applet online—v. 1	Study 1: Online survey data	Reliance on area models to compare fractions Low utilization of pedagogical tools Unaware of differentiated strategies between groups of fractions	Addition of a demonstration video to illustrate how the various tools could be used to scaffold thinking
Study 2	Face-to-face and online follow-up for in-service teachers (in-class introduction to applet) Applet online—v. 2 with video and text summaries	Study 2: Online survey data	Fewer participants relied on area models to compare fractions, although some still did Videos were viewed by about half of the participants. Those who viewed the videos used more tools and more varied strategies	Inclusion of interval marks feature Use of interactive whiteboard to enhance record of markup notations Inclusion of active reflection questions

**Figure 2.** Iterative process of design and research enactments.

### Study 1

The applet was first enacted in a hybrid (blended) class of 28 preservice elementary teachers enrolled in a mathematics methods course at a large southwestern university. The students were introduced to the applet via an

online assignment. They were first asked to read a chapter about various fraction representations in their textbook (Cathcart, Pothier, Vance, & Bezuk, 2003) and to discuss it in an online discussion board. Next, they were asked to explore the number-line applet sequence online. It is important to note that the readings they were assigned focused on area models to represent and compare fractions. Thus, the use of number-line strategies represented a slightly different approach. When they had finished working with the applet, they were asked to complete the survey included in Appendix A of the textbook.

The *Results* section of this report contains quantitative and qualitative statistics from this study juxtaposed with the results from Study 2 in order to illustrate how changes in applet enactment elicited large shifts in how the various pedagogical features of the applet were experienced. Although the applet was initially designed to be a didactic object to support in-class communications, the design team thought it would be useful to try it out with this group of students. The results from Study 1 indicate that while a few of the students availed themselves of the pedagogical features in the ways intended, the majority of the respondents used only one strategy: the  $\frac{1}{2}$  benchmark. Moreover, 5 out of 28 respondents reported that they converted the fractions to decimals or percents, or used area models (such as pie diagrams) to compare the fractions before placing them on the number line. As noted earlier, this was predictable given their reading assignment and lack of in-class discussion prior to enactment. There were also several other unintended interpretations, such as the fact that some students thought they would be cheating if they moved the fractions after revealing an answer, and others thought that they were close enough and therefore did not have to read the summaries that described an alternative method to the one they had used.

Although the design team had hypothesized that the interface features were not self-evident and the overall approach of using an interactive applet as a learning tool as opposed to a tutorial tool was not a practice in which they had engaged before, the specific insights from the comments emerged from Study 1. These insights provided specific goals for making an overview video tutorial that would, it was hoped, serve two purposes: (a) It would bridge the gap between how the in-class discussion relates to the assigned as homework, and (b) it would serve as an orientation to guide students who were absent when the applet was used as a didactic object in class. The design team concluded its reflection on Study 1 by resolving to create an overview video that would demonstrate the intent of the activity, the role of each of the pedagogical features, and a discussion of the im-

portance of developing a repertoire of possible strategies depending on the numbers at hand. The design team imagined that the video would be consistent with the third design feature from PDC-DF: Teacher (or capable student) should model high-level performance.

## Study 2

The second group of participants was comprised of 20 in-service upper elementary teachers (grades 3-5) enrolled in the second year of a 2-year Mathematics Specialist Certificate program. The course focused on developing teachers' pedagogical content knowledge regarding the ways in which conceptual thinking (with an eye toward algebraic ideas) can be infused into activities in the upper primary grades. The hybrid course was designed so that participants alternated between bimonthly face-to-face meetings (in 3-hour sessions held after school) and online sessions that involved approximately three hours of work. The online sessions generally followed a pattern of a posted reading that described some pedagogy and mathematical content, an extended mathematical investigation (of which the number-line series is an example), a "try it on" exercise in which participants were expected to use an activity in their own classrooms, and a forum discussion or journal entry that involved a reflection on the online unit. It is important to note that the goal for the online content was not to introduce new material, but rather to provide activities that would engage the participants in a review and deeper exploration of the content discussed during the face-to-face sessions. Thus, the number-line applet was one of many exploratory applets that first served as didactic objects during a face-to-face session, and then was used to support further exploration as an online homework assignment.

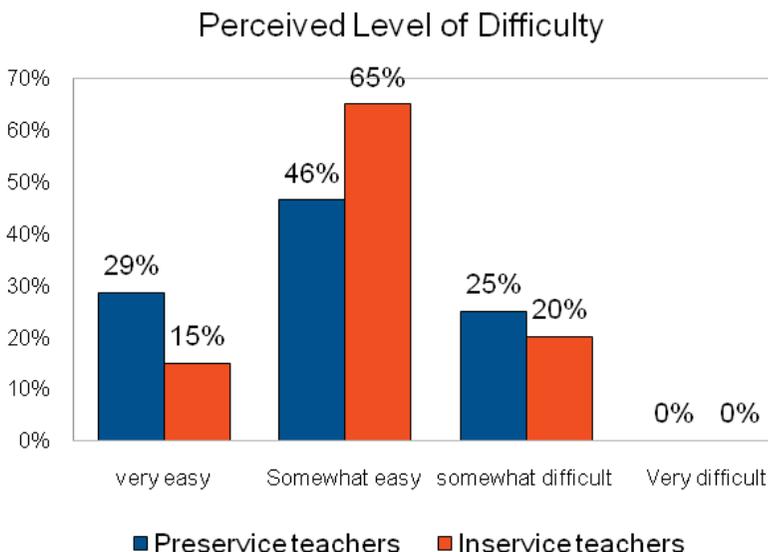
### COMPARING RESULTS FROM STUDY 1 AND STUDY 2

It may appear disingenuous to compare the results from the two studies because they were enacted in different settings and the two audiences had vastly different agendas and background experiences. We do so merely to view which features had greatest differential effect, and which appeared impervious to differing circumstances or social settings.

### Question 1: Did the participants find the number-line activity to be cognitively difficult?

Stein et al. (2000) did not necessarily equate or determine cognitive demand based on students' perception of the difficulty of a task. However, in order to determine if the participants generally felt that the task was engaging, the questionnaire included a number of questions regarding the perceived difficulty as well as questions about whether different linear-based reasoning strategies were used.

Question 1 asked the respondents to first rate the level of difficulty for the task (1 = *very easy*; 4 = *very difficult*); and then to elaborate on the rating in a free-response field. The results of this question are shown in Figure 3. As indicated in Figure 3, while 20% of in-service teachers did report the task to be somewhat difficult, the majority of respondents from both groups found the number line task to be somewhat easy. This finding was particularly surprising to the instructors of the in-service class because when they had assigned these activities to other participants in earlier seminars (using paper and pencil), the teachers indicated that they had a great deal of difficulty and they lacked confidence in how to choose a most expedient strategy. Thus the design team set to determine whether and how the level of cognitive demand might have declined during the online enactment.



**Figure 3.** Perceived level of difficulty for number line applet.

One potential hypothesis for the perceived low level of difficulty is that only 54% of the in-service teachers and only 62% of the preservice teachers reported using number-line strategies to solve the tasks. For example, some preservice teachers reported using different conversion strategies, such as changing the fractions to decimals or percents (several of them even admitted using a calculator). Others, not surprisingly given their assigned reading, reported drawing area models. For example, one preservice student commented, "It would be nice to have models or sketches of the fractions as a visual—I am sick of making them myself!"

In contrast, none of the in-service teachers reported using any conversion methods. In fact, one teacher wrote, "I thought the point of the activity was to deal with fractions so I dealt with them as fractions only." Another explained that although she relied on the  $\frac{1}{2}$  benchmark, she used a sophisticated linear strategy: "I used the quantities as fractions. I found out what was half of the denominator." The implication from this set of responses indicates clearly that the use of the applet in class (along with the instructor's emphasis on using linear strategies) enabled the establishment of a socially mediated math practice: When solving these number-line problems, the goal is to think about how the various numerators relate to the denominators.

A third potential source of data to address the question of how the cognitive demand may have decreased when the activity was enacted online was whether or not the participants attempted to investigate the challenge task of ordering  $\frac{1}{x}$ ,  $\frac{2}{x}$ , and  $\frac{3}{x}$ . Results from the survey reveal that only 19% of the preservice and only 12% of the in-service teachers reported attempting this set of problems. Some of the preservice teachers indicated that they assumed that the default value, 5, was acceptable or not changeable and therefore found the task to be easy because they could use a common denominator strategy. None of the in-service teachers indicated that they were confused by the interface, but most simply stated that they did not attempt the problem.

## **Question 2: Role of Pedagogical Features in Maintaining Cognitive Demand**

The question of which pedagogical features (as represented in Figure 1 and Figure 2) were perceived as effective in maintaining cognitive demand was assessed by counting the number of respondents who used each feature and reading their open-ended comments for further clarification. The results of these data are compiled in Table 3.

**Table 3**  
Percentage of Teachers Indicating Use of Each Pedagogical Feature

Pedagogical feature	Teachers indicating feature was helpful (%)	
	Preservice	In-service
Feature 1: Show $\frac{1}{2}$ benchmark	82%	65%
Feature 2: Reveal one answer and adjust placements	57%	70%
Feature 3: Use of only nonjudgmental feedback	61%	60%
Feature 4: Reflection and strategy discussions	57%	40%
Feature 5: Use of various strategies based on numbers	71%	95%

*Feature 1: Show  $\frac{1}{2}$  benchmark.* This feature was designed to scaffold users who were not sure about what strategy to use. As presented in Table 3, 82% of the preservice teachers and 65% of the in-service teachers used this feature. The open-ended questions reveal a slightly different story: Almost all respondents in both groups did use the  $\frac{1}{2}$  benchmark as a strategy, and many noted that they used it exclusively. They simply did not click on the button. For example, one preservice teacher described its utility by saying, “It was nice to have the  $\frac{1}{2}$  button because it gave me a visual of where to start if the fractions were close to  $\frac{1}{2}$ .” However, some explained that they did not always use the feature because it seemed “self-evident.” For example, one preservice teacher wrote, “I didn’t need it.  $\frac{1}{2}$  is a pretty obvious location, so I didn’t feel it was necessary.” Another in-service teacher wrote, “I thought it would be TOO helpful. . . [I] wanted to see what I could do without that support.”

*Feature 2: Reveal answers individually.* The difference between the percentage of pre- versus in-service teachers using the reveal-one-at-a-time strategy is somewhat striking. From a social point of view, this could be interpreted to indicate that the teachers in the face-to-face PD sessions developed a social norm for using feedback to revise answers (i.e., developing the

habit of mind to use the computer as an exploratory microworld). In contrast, most preservice teachers did not seem to view their activity in terms of interacting with the technology to learn in an exploratory manner. For example, one preservice student wrote, "I would have preferred one 'show all answers' button. . . it felt like cheating if I changed one after viewing a correct answer." From a social perspective, comments such as this along with others suggesting the need for a more expeditious show all answers button, seem to give some indication that preservice teachers, who are still college students, may view their in-school practices as work leading to assessment, rather than work leading to learning. This hypothesis is supported by the fact that although some of the in-service teachers mentioned that they would have liked a "reveal all" button, many others indicated that seeing one at a time was experienced as more pedagogically rich. For example, one teacher wrote, "I liked viewing the answers one at a time because I could focus on how far off I was from the exact point. On one problem I moved my answers to be on the exact point. If I had revealed all at once, I wouldn't have bothered moving the others."

*Feature 3: Use of nonjudgmental feedback.* One of the most critical differences between the number-line applet series and online tutorials is the deliberate use of nonjudgmental feedback. When a user click to see the correct answers, the actual placement of each fractional quantity is shown, but no judgment is made assessing whether the user's answer is correct. This determination is purposely left to each individual teacher participant. The design group suspected that answers to this survey question would align with those regarding the reveal feature. Surprisingly, they did not. Instead, both groups interpreted this feature in more affective terms. While approximately 60% of each group recognized the learning potential from analyzing their own results, the remaining 40% of both groups felt that more personalized feedback saying that an answer was too far away would have been more motivating or personally encouraging. For example, one preservice teacher wrote:

I think more personal feedback would be more helpful in making the simulation more fun. By getting feedback, the program is more like a game and would motivate students to think about the concise positions of the fraction in order to get a positive response from the computer. I would like to see answers to reflection questions as well, but maybe at the end of the program, so the reader can think about it before, and then see answers later.

*Feature 4: Inclusion of written reflection and summary questions.* As the results in Table 1 indicate, neither group found this feature particularly useful. Although 57% of the preservice teachers claimed they read them,

most stated that they were not helpful, or they only skimmed the text because it was not visually appealing to read. Comments from the 60% of the in-service teachers who did not read the text generally indicated they believed they had already talked about these strategies in the face-to-face session. This rather dismal result suggest that simple text is not effective in an online setting. One respondent stated that she might have read them if she were forced to—that is, if the program would not advance until the paragraph was at least shown and some interactive quiz was included and completed.

*Feature 5: Use of various strategies based on purposeful sequencing.* The results from this item indicate that 95% of in-service teachers recognized that the fractions were grouped in clusters that lent themselves to specific strategies. In contrast, only 71% of the preservice teachers noticed this. This is a fascinating finding, and one that the instructor of the preservice course pointed out during the ensuing debriefing period.

*Feature 6: The inclusion of a video.* The video overview was included to orient those in-service teachers who were not present at the in-class discussion session. When creating the number-line video, the narrator combined a demonstration of how to use the interface tools with a description of the types of thinking and explanations that were envisioned. If this video was effective, then one would expect those who chose to view it to have used more tools more often than those who chose not to view it. Survey results indicate that half of the in-service teachers chose to view the video. As represented in Figure 4, of those 10, 70% reported using both scaffolding tools while only 30% of those participants who did *not* watch the video used both tools. This could indicate that the video did serve to maintain the cognitive demand of the task, or it could indicate that those teachers who did not want to watch the video already had a strategy in mind and did not deviate from it.

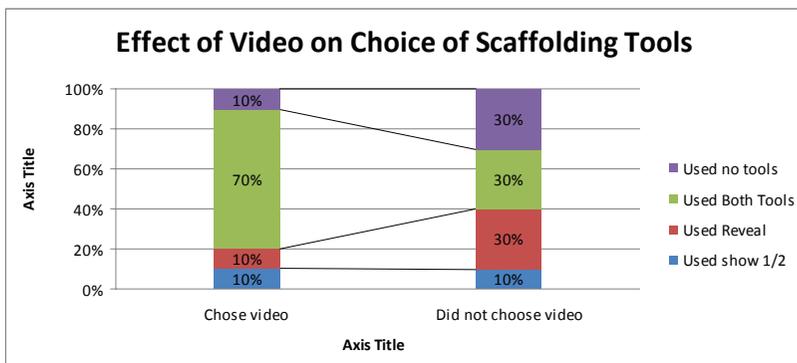


Figure 4. Relations between video choice and use of specific tools.

### **Question 3: Would Teachers Use this Applet in their Own Classrooms?**

Of the 28 preservice teachers, 25 said that they would like to use this activity in their own classrooms. All of these respondents stated that they would first demonstrate using a projector, and then assign students seat-work, either individually or in pairs. Interestingly, many of the preservice teachers also mentioned formal assessment would have to be incorporated before they would use it with students. For example, one wrote, "I would only use it if the students were scored on their placements BEFORE revealing the answers (which would be good immediate feedback for them)." One interpretation of this statement is that these respondents did not perceive the applet as providing an opportunity to engage in exploration or support conversation; it was seen as a tutorial and would, if not further discussed, be enacted in classrooms as such.

Of the 85% of in-service teachers who stated that they would use the tool, two mentioned that they would use an interactive whiteboard, and three mentioned that they would focus on strategies. For example, one fourth-grade teacher wrote, "It would be good for students to work on the problems in small group. Then have a whole-class discussion where students are showing how they solved the problem." These comments indicate that some of the teachers were imagining engaging in social practices that leverage the affordances of new technologies.

## **DISCUSSION**

The results from these two iterations of design research focused on how a series of cognitively demanding number-line activities could be enhanced by the use of technology. Given the design team's view that learning is a social process mediated by conversation tools and engagement practices, it seemed most expedient to complement the Mathematics Task Framework (Stein et al., 2000) with applet design principles from the IDEA framework (Underwood et al., 2005). The design goal was to create computer-based versions of activities that had previously been realized in face-to-face sessions by challenging the instructors to think about what pedagogical moves they enacted in prior implementations that maintained cognitive demand, and what new computer affordances could be added to ensure that high cognitive demand could be maintained when users enacted the applet online. The accompanying research goal was to determine the degree to which the various interface features served to maintain cognitive demand as the teach-

er participants worked with the applets on their own. The following discussion summarizes the findings from this research. Each section describes specific findings that can inform teacher educators aiming to support the use of technologies. In particular, the goal is to support prospective and practicing teachers who are often asked to use newly purchased equipment, such as computer projectors and interactive whiteboards, without being given much pedagogical guidance and research-based findings regarding how to leverage various aspects of these interactive technologies.

### **Recommendation 1: Pedagogical Moves that Maintain Cognitive Demand in Classrooms Provide Rich Starting Points for Designing Technology-based Activities**

The results of the online surveys indicated that the interface features that were inspired by efforts to make cybernetic instantiations of the instructors' in-class pedagogical moves (e.g., the show  $\frac{1}{2}$  and reveal one at a time buttons) were generally effective for maintaining cognitive demand. For example, even though they could have revealed all of the fractions at once, a large majority of the in-service teachers chose to reveal just one at a time so they could readjust the remainder of their answers and reorganize their thinking just as the instructors in other face-to-face sessions have done. In this way, the computer interface, like the teacher, was able to provide a way to encourage multiple approaches to arrive at an initial answer and also encouraged users to consider multiple strategies for placing the fraction quantities depending on the quantities presented in the task.

In contrast, fewer of the preservice teachers enacted these pedagogical features in this way. This could be explained by the fact that they did not engage in the use of the applet as a didactic object prior to working with it on their own, or by the fact that they lacked teaching experience. The implication for teachers and teacher educators is that one should choose activities that support conversation-based pedagogical moves, such as encouraging discussions of multiple strategies. If the applets are not first introduced in a whole-class setting, then the use of video seems to be more engaging than the use of static text to engage students in the desired enactment processes and affective reactions.

### **Recommendation 2: Nonjudgmental Feedback Promotes Deeper Conceptual Reorganization than Judgmental Scoring**

As indicated by the PDC-DF (see Table 1), there are several powerful connections between Stein et al.'s (2000) factors for maintaining cognitive demand and Underwood et al.'s (2005) IDEA applet design principles. In particular, Stein et al. suggested providing students with means for monitoring their own progress and Underwood et al. suggested providing appropriate status feedback. One goal of this research was to explore what appropriate cybernetic status feedback might be, and the degree to which it enabled students to monitor their own progress.

The results from these surveys were mixed. On the one hand, some of the respondents stated that the amount of feedback was provocative and they recognized the value of this approach as a way to prompt explanations and justifications. On the other hand, roughly 40% of each group preferred to have judgmental scoring. These respondents argued from an affective perspective: They claimed that the extra cognitive effort was neither as satisfying nor as motivating as having the computer determine the correctness of an answer. This is a critical finding because the users' voices must be heard, and yet causes a conundrum for designers.

The original hypothesis was that deep thinking shifts the locus of learning from the teacher to the student, which is essential in online learning settings. One possible resolution to this scoring conundrum might be to include some type of range tolerance scale that users could set. The computer would be instructed to score based on the user's choice for how close each fraction placement has to be. One implication, based on a social view of learning, is that teachers must focus on engaging students in mathematical practices that involve judging their own work so that students strive for the internal satisfaction of *understanding* rather than the external satisfaction of confirmation from an outside arbiter.

### **Recommendation 3: Applets Should be Assessed in Terms of Ability to Serve as Didactic Objects**

There is presently a scramble in the curriculum publishing world to create online activities and interactive whiteboard lessons (IWBs). These production efforts are directed by studies claiming that IWBs increase student attention (Marzano & Haystead, 2009). Lerman and Zevenbergen (2007) conducted classroom-based research and concluded that although attention

may be increased, the level of questioning is actually decreased. They concluded

[T]he technologically impressive features of the IWB can lead to it being used to close down further the possibility of rich communications and interactions in the classroom as teachers are seduced by the IWB's ability to capture pupils' attention. We suspect, also, that teachers' advance preparation for using the IWB, often via the ubiquitous PowerPoint package or pre-prepared lessons for the IWB, are leading to a decreased likelihood that teachers will deviate in response to pupils' needs and indeed might notice pupils' needs less frequently through the possibility to increase the pacing of mathematics lessons (p. 3-175).

These observations indicate that when enacted, prepackaged IWB materials most likely lose cognitive demand. The implication from this finding is that teacher educators need to help pre- and in-service teachers seek and develop interactive activities (rather than static PowerPoint slides), that possess the most potential to press students for justifications, explanations, and/or meaning. Results from the surveys indicate that even though they were not required to check their answers, all respondents did so and many used the tools within the environments to retest their revised thinking strategies. The broad range of entry points and the potential for reorganizing one's thinking is more likely to occur if the ideas being examined are rich and ripe for explanations that can resolve perturbations. Like Thompson's (1994) Over and Back program, it makes sense to find interactive applets that create surprise and encourage resolution.

These ideas, which are consistent with Stein et al.'s (2000) mathematical task framework, suggest the following implications for teachers and teacher educators designing and choosing interactive didactic objects:

1. Activities should include multiple entry points and scaffold tools so that conversations can focus on those that are *most expedient, or most explanatory, or most sophisticated*.
2. Activities should be enacted in ways that give students the chance to make predictions, commit to them, and examine outcomes (Underwood et al., 2005) so the individuals and class serve as arbiters of correct answers rather than an outside source, such as the computer or the teacher.

#### **Recommendation 4: Reduce Factors that Diminish Cognitive Demand**

This study identified several factors that may have contributed to diminishing cognitive demand. First, users' activity was not graded or recorded. Open-ended comments from both groups indicated that they would have put more effort into the task had their results been measured. This is consistent with the users' comments regarding the need for more judgmental feedback that would motivate them to beat the high score.

A second factor that might have reduced cognitive demand was that some users reported feeling more comfortable with area models (e.g., pie or shaded rectangles). Unlike face-to-face settings in which the teacher can probe for specific *types* of explanations, the online environment cannot compel users to think in specific ways. A third, related issue was the lack of conceptual explanations that were required or shared. Although the visual feedback may have invited self-explanations, it might be useful to add mock student explanations and ask participants to rank or grade the explanations. In this way, the computer does compel the users to at least *think about* and respond to alternative or novel explanations much like those teachers elicit during rich in-class discussions. One implication for teachers and teacher educators is that activities should allow students to offload lower cognitively demanding tasks (which aligns with Factor 6 of the PDC-DF, as presented in Table 1). For example, later revisions of the number-line applet contain a "make segment marks" button that divides the number line into various subintervals. A new design-research cycle is currently being conducted to determine if this feature will enhance conversations by ramping up the ability to compare the relative sophistication of different approaches.

The key to supporting fidelity between how a task is envisioned and how it is enacted (either online or in person) lies in affecting social practices, values, and judgments that emerge as the students engage in the activity. By developing and enacting further interactive didactic objects that support commognition (Sfard, 2008), we can further support the development of technology-based habits of mind wherein computer tools are seen as exploratory microworlds rather than superficial tutorials in which moving or revising an answer is seen as cheating.

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